Chapter-8	Application of Integrals समाकलनों के अनुप्रयोग

Q 1 **Find the area of the circle $x^2+y^2=a^2$ by integration.**

वृत्त $x^2+y^2=a^2$ का क्षेत्रफल समाकलन द्वारा निकालें।

Q 2 **Find the area of the smaller portion of the circle $x^2+y^2=a^2$ cut off by the line $x=\frac{a}{2}$.**

सरल रेखा $x=\frac{a}{2}$ द्वारा विभाजित वृत्त $x^2+y^2=a^2$ के भागों में से छोटे भाग का क्षेत्रफल निकालें।

Q 3 **Find the area of the region bounded by the parabola $y=x^2$ and $y=|x|$.**

परवलय $y=x^2$ तथा $y=|x|$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात करें।

Q 4 **Find the area bounded by the parabola $y^2=4x$ and the straight line $x+y=3$.**

परवलय $y^2=4x$ तथा सरल रेखा $x+y=3$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात करें।

Q 5 **Find the area of the region bounded by the parabolas $x^2=y$ and $y^2=x$.**

परवलयों $x^2=y$ तथा $y^2=x$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात करें।

Q 6 **Find the area lying above the x-axis and included between the curves $x^2+y^2=8x$ and $y^2=4x$.**

x-अक्ष के ऊपर वृत्त $x^2+y^2=8x$ एवं परवलय $y^2=4x$ के मध्यवर्ती क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

Q 7 **Find the area of the region enclosed by the curves $x^2+y^2=1$ and $(x-1)^2+y^2=1$.**

वक्रों $x^2+y^2=1$ तथा $(x-1)^2+y^2=1$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

Q 8 **Using integration, find the area of $\triangle ABC$ whose vertices are A(2,0), B(4,5) and C(6,3).**

समाकलन विधि का प्रयोग करते हुए एक ऐसे त्रिभुज ABC का क्षेत्रफल ज्ञात कीजिए जिसके शीर्ष A(2,0), B(4,5) एवं C(6,3) हैं।

Q 9 **Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$.**

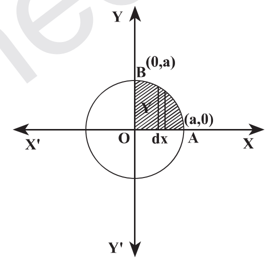
दीर्घ वृत्त $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ एवं रेखा $\frac{x}{a}+\frac{y}{b}=1$ से घिरे लघु क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

5- Marks Solution

1 Ans : -

Whole area enclosed by the given circle

=4(area of the region AOBA bounded by the curve , x-axis and the ordinates x=0 and x=a)



$$= 4\int_0^a y\,dx$$

$$= 4\int_0^a \sqrt{a^2-x^2}\,dx$$

$$= 4\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

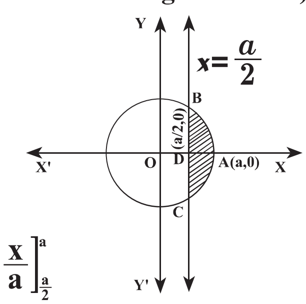
$$= 4\left[\left(\frac{a}{2}\times 0+\frac{a^2}{2}\sin^{-1}1\right)-0\right]$$

$$= 4\left(\frac{a^2}{2}\right)\left(\frac{\pi}{2}\right)=\pi a^2 \text{ sq. units}$$

2 Ans : -

From the figure ,

Required area = 2(area of the region ABDA)



$$= 2\int_{\frac{a}{2}}^{a} y\,dx$$

$$= 2\int_{\frac{a}{2}}^{a} \sqrt{a^2-x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{2}}^{a}$$

$$= 2\left[\frac{a}{2}\times 0-\frac{a}{4}\times\frac{\sqrt{3a^2}}{2}+\frac{a^2}{2}\left(\sin^{-1}1-\sin^{-1}\frac{1}{2}\right)\right]$$

$$= 2\left[\frac{-\sqrt{3}}{8}a^2+\frac{a^2}{2}\left(\frac{\pi}{2}-\frac{\pi}{6}\right)\right]$$

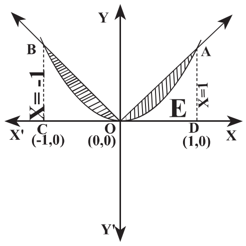
$$= 2\left[\frac{-\sqrt{3}}{8}a^2+\frac{a^2}{2}\times\frac{\pi}{3}\right]=a^2\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right] \text{ sq. units}$$

3 Ans : -

Given curves are

$x^2 = y$ ....(1)

and $y = |x|$ ....(2)



From (2)

$$y = |x| = \begin{cases} x \text{ , when } x \geq 0 \\ -x \text{ , when } x < 0 \end{cases}$$

on solving (1) and (2), we get

curve $x^2 = y$ and $y = x$ intersect each other at O (0,0) and A (1,1).

and curve $x^2 = y$ and $y = -x$ intersect each other at O (0,0) and B(- 1,1).

$\therefore$ shaded region is the required region .

Draw AD $\perp$ OX and BC $\perp$ OX'.

$\therefore$ Required area = 2(area of region OEAO)

$$= 2\int_0^1 (y_1 - y_2)dx$$

$$= 2\int_0^1 ydx(\text{for the line OA}) - 2\int_0^1 ydx(\text{for the parabola})$$

$$= 2\int_0^1 xdx - 2\int_0^1 x^2dx = 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^3}{3}\right]_0^1$$

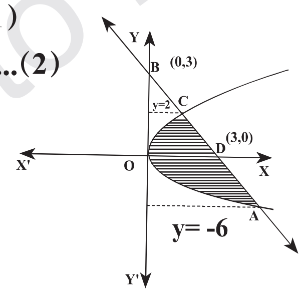
$$= 2\left(\frac{1}{2} - 0\right) - 2\left(\frac{1}{3} - 0\right) = 1 - \frac{2}{3} = \frac{1}{3} \text{ sq. units}$$

4 Ans : -

Given curves are

$y^2 = 4x$ ....(1)

and $x + y = 3$ ....(2)



From (1) and (2)

$y^2 = 4(3 - y)$

$\Rightarrow y^2 + 4y - 12 = 0$

$\Rightarrow y = -6 , 2$

$\therefore$ Area of shaded region

i.e. Area of region OCDAO

$$= \int_{-6}^{2} (x_1 - x_2)dy = \int_{-6}^{2}\left[(3 - y) - \frac{y^2}{4}\right]dy$$

$$= \left[3y - \frac{y^2}{2} - \frac{y^3}{12}\right]_{-6}^{2} = \left(6 - 2 - \frac{8}{12}\right) - \left(-18 - 18 - \frac{-216}{12}\right)$$

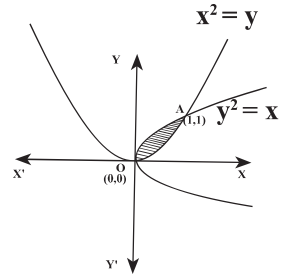
$$= \left(\frac{10}{3} + 18\right) = \frac{64}{3} \text{ sq. units}$$

5 Ans ; -

Given parabolas are

$x^2 = y$ ....(1)

and $y^2 = x$ ....(2)



on solving (1) and (2)

points of intersection are O(0,0) and A(1,1).

$\therefore$ Required area

$$= \int_0^1 (y_1 - y_2)dx = \int_0^1 (\sqrt{x} - x^2)dx$$

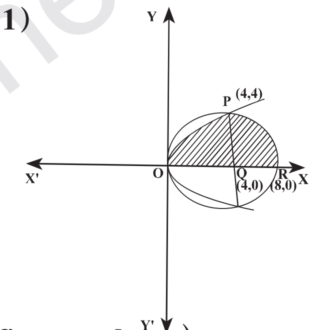
$$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \text{ sq. units}$$

6 Ans : -

Given curves are

$x^2 + y^2 = 8x$ ....(1)

and $y^2 = 4x$ ....(2)



From (1) and (2)

$x^2 + 4x = 8x$

or , $x^2 - 4x = 0 \Rightarrow x = 0,4$

$\therefore$ When x = 0 then y = 0

When x = 4 then y = 4(in first quadrant)

$\therefore$ Area of regionOPRQO =

Area of regionOPQO + Area of regionQPRQ

$$= \int_0^4 ydx(\text{for parabola}) + \int_4^8 ydx(\text{for circle})$$

$$= \int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{8x - x^2}\,dx$$

$$= 2\int_0^4 \sqrt{x}\,dx + \int_4^8 \sqrt{4^2 - (x-4)^2}\,dx$$

$$= 2\left[\frac{2}{3}x^{3/2}\right]_0^4 + \int_0^4 \sqrt{4^2 - t^2}\,dt \qquad \left(\begin{array}{l}\text{Put } x - 4 = t \\ \therefore dx = dt \\ \text{if } x = 4 , t = 0 \\ \text{if } x = 8 , t = 4\end{array}\right)$$

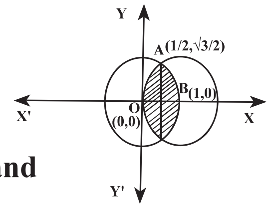
$$= \frac{4}{3}(4^{3/2} - 0) + \left[\frac{t}{2}\sqrt{4^2 - t^2} + \frac{16}{2}\sin^{-1}\frac{t}{4}\right]_0^4$$

$$= \frac{4}{3} \times 8 + [(0 + 8\sin^{-1}1) - (0 + 8\sin^{-1}0)]$$

$$= \frac{32}{3} + \frac{8\pi}{2} - 0 = \frac{32}{3} + 4\pi$$

$$= \frac{4}{3}(8 + 3\pi) \text{ sq. units}$$

7 Ans



Given circles are

$$x^2 + y^2 = 1 \quad ....(1)$$

and $(x-1)^2 + y^2 = 1 \quad ....(2)$

centre of circle(1) is (0,0) and radius = 1 unit

centre of (2) is (1,0) and radius = 1unit

by (1) − (2), we get $2x - 1 = 0$

$$\Rightarrow x = \frac{1}{2}$$

$\therefore$ **From (1), when** $x = \frac{1}{2}$, $y^2 = \frac{3}{4}$

$$\therefore y = \pm\frac{\sqrt{3}}{2}$$

$\therefore$ **Required area = 2 × area of regionOABO**

$$= 2\int_0^{\sqrt{3}/2} (x_1 - x_2)dy$$

$$= 2\int_0^{\sqrt{3}/2} \left[\sqrt{1-y^2} - \left(1 - \sqrt{1-y^2}\right)\right]dy$$

$$\begin{bmatrix} \text{From eqn}(1),\ x = \pm\sqrt{1-y^2}, \therefore \text{ For arc AB } x = \sqrt{1-y^2} \\ \text{From eqn}(2),\ x-1 = \pm\sqrt{1-y^2}, \therefore \text{ For arc OA } x = 1-\sqrt{1-y^2} \end{bmatrix}$$

$$= 4\int_0^{\sqrt{3}/2} \sqrt{1-y^2}\,dy - 2\int_0^{\sqrt{3}/2} 1dy$$

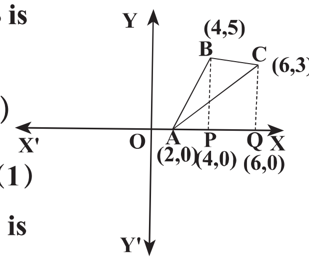
$$= 4\left[\frac{y}{2}\sqrt{1-y^2} + \frac{1}{2}\sin^{-1}y\right]_0^{\sqrt{3}/2} - 2[y]_0^{\sqrt{3}/2}$$

$$= 2\left[\frac{\sqrt{3}}{2}\cdot\frac{1}{2} + \sin^{-1}\frac{\sqrt{3}}{2}\right] - 0 - 2\left(\frac{\sqrt{3}}{2} - 0\right)$$

$$= \frac{\sqrt{3}}{2} + \frac{2\pi}{3} - \sqrt{3}$$

$$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) \text{ sq. units}$$

8 Ans : -



Equation of side AB is

$$y - 0 = \frac{5-0}{4-2}(x-2)$$

or $y = \frac{5}{2}(x-2) \quad ....(1)$

Equation of side BC is

$$y - 5 = \frac{3-5}{6-4}(x-4)$$

or , $y - 5 = \frac{-2}{2}(x-4)$

or , $y - 5 = -x + 4$ **or ,** $y = -x + 9 \quad ....(2)$

Equation of side AC is

$$y - 0 = \frac{3-0}{6-2}(x-2)$$

or , $y = \frac{3}{4}(x-2) \quad ....(3)$

From B , draw BP ⊥ OX and from C draw CQ ⊥ OX.

$\therefore$ Area of $\triangle$ ABC = ar($\triangle$APB) + ar(trapezium BPQC) - ar($\triangle$AQC)

$$= \int_2^4 y_{AB}dx + \int_4^6 y_{BC}dx - \int_2^6 y_{AC}dx$$

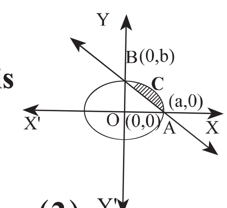
$$= \int_2^4 \frac{5}{2}(x-2)dx + \int_4^6 (-x+9)dx - \int_2^6 \frac{3}{4}(x-2)dx$$

$$= \frac{5}{2}\left[\frac{x^2}{2} - 2x\right]_2^4 + \left[-\frac{x^2}{2} + 9x\right]_4^6 - \frac{3}{4}\left[\frac{x^2}{2} - 2x\right]_2^6$$

$$= \frac{5}{2}[0+2] + [36-28] - \frac{3}{4}[6-(-2)]$$

$$= 5 + 8 - 6 = 7 \text{ sq. units}$$

9. Ans



Given equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad ....(1)$$

and line is $\frac{x}{a} + \frac{y}{b} = 1 \quad ....(2)$

we have to find the area of shaded region

$\therefore$ **Area of region ABCA** $= \int_0^a (y_1 - y_2)dx$

$$= \int_0^a ydx(\text{for ellipse}) - \int_0^a ydx(\text{for line})$$

$$= \int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx - \int_0^a \frac{b(a-x)}{a}dx$$

$$= \frac{b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a - \frac{b}{a}\left[ax - \frac{x^2}{2}\right]_0^a$$

$$= \frac{ab}{2}\left(\sin^{-1}1 - \sin^{-1}0\right) - \left(ab - \frac{ab}{2}\right)$$

$$= \left(\frac{\pi ab}{4} - \frac{ab}{2}\right) \text{ sq. unit}$$

$\therefore$ **Required area** $= \left(\frac{\pi ab}{4} - \frac{ab}{2}\right)$**sq.unit**

Chapter-9	Differential Equation अवकल समीकरण

MCQ:- (बहुविकल्पीय प्रश्न) -

Q 1 The order of the differential equation

$2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ **is**

अवकल समीकरण $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ की कोटि है:

(a) 2 (b) 1

(c) 0 (d) Not defined (अपरिभाषित)

Q 2 The differential equation representing the family of curve $y^2=2c(x+\sqrt{c})$, where c is a positive parameter, is of

$y^2=2c(x+\sqrt{c})$, जहाँ C एक घनात्मक प्राचल है,से निरूपित होने वाले वक्र–कुल का अवकल समीकरण निम्नांकित में किस प्रकार का होगा?

(a) order 1,degree 3 (कोटि 1) (घात 3) (b) order 2,degree 3 (कोटि 2) (घात 3)

(c) order 3,degree 3 (कोटि 3) (घात 3) (d) order 4,degree 4 (कोटि 4) (घात 4)

Q 3 A solution of the differential equation

$\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}+y=0$ **is.**

अवकल समीकरण $\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}+y=0$ का समाधान है:

(a) y = 2 (b) y = 2x

(c) y = 2x-4 (d) $y = 2x^2-4$

Q 4 The order of the differential equation

$\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ **is**

अवकल समीकरण $\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ की कोटि है:

(a) 1 (b) 2

(c) 3 (d) 4

Q 5 The degree of the differential equation

$\left(\frac{d^3y}{dx^3}\right)^2-3\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^4=y^4$ **is**

अवकल समीकरण $\left(\frac{d^3y}{dx^3}\right)^2-3\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^4=y^4$ का घात है :

(a) 4 (b) 2

(c) 3 (d) 1

Q 6 The order and degree of the differential equation $\frac{d^4y}{dx^4}=y+\left(\frac{dy}{dx}\right)^4$ are respectively

अवकल समीकरण $\frac{d^4y}{dx^4}=y+\left(\frac{dy}{dx}\right)^4$ की कोटि एवं घात क्रमशः है:

(a) 2,2 (b) 4,1

(c) 2,4 (d) 4,2

Q 7 Integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{1+y}{x}$ is

अवकल समीकरण $\frac{dy}{dx}+y=\frac{1+y}{x}$ का समाकलन गुणांक है:

(a) e^x (b) xe^x

(c) e^x/x (d) x/e^x

Q 8 What is the integrating factor of

$\frac{dy}{dx}+y\sec x=\tan x$

$\frac{dy}{dx}+y\sec x=\tan x$ का समाकलन गुणांक क्या है?

(a) secx+tanx (b) log(secx+tanx)

(c) $e^{\sec x}$ (d) secx

Q 9 The general solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}$ is

अवकल समीकरण $\frac{dy}{dx}=\frac{y}{x}$ का व्यापक हल है:

(a) y=k/x (b) y=kx

(c) y=klogx (d) logy=kx

Q 10 If a and b are the order and degree of the differential equation

$y\frac{dy}{dx}+x^3\left(\frac{d^2y}{dx^2}\right)+xy=\cos x$ **, then**

यदि a तथा b अवकल समीकरण

$y\frac{dy}{dx}+x^3\left(\frac{d^2y}{dx^2}\right)+xy=\cos x$

के कोटि तथा घात हैं, तो

(a) a<b (b) a=b

(c) a>b (d) None of these (इनमें से कोई नहीं)

Very Short Question (2 Marks)

Q 1 Find the differential equation of the family of curves $x^2+y^2=2ax$.

वक्रों के कुल $x^2+y^2=2ax$ का अवकल समीकरण ज्ञात कीजिए।

Q 2 Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

ऐसे परवलयो के कुल का अवकल समीकरण निर्मित कीजिए ,जिनका शीर्ष मूल-बिन्दु पर है और जिनका अक्ष धनात्मक y-अक्ष की दिशा में है।

Solve the following differential equations : -

निम्नलिखित अवकल समीकरणों को हल करें : -

Q 3 $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

Q 4 $\frac{dy}{dx}=\sqrt{4-y^2}\ ,-2<y<2$

Q 5 $\frac{dy}{dx}=\frac{x-1}{y+2}\left(y\neq -2\right)$

Q 6 $\frac{dy}{dx}=1-x+y-xy$

Q 7 $\frac{dy}{dx}=(1+x^2)(1+y^2)$

Q 8 $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$

Q 9 $x\frac{dy}{dx}-y=x^2$

Q 10 $x\frac{dy}{dx}=x+y$

Short Question (3 Marks)

Solve ; - (हल करें : -)

Q 1 $\sec^2x\tan y dx+\sec^2y\tan x dy=0$

Q 2 $y\log y dx-x dy=0$

Q 3 $e^x\tan y dx+(1-e^x)\sec^2y dy=0$

Q 4 $x dy-y dx=\sqrt{x^2+y^2}\,dx$

Q 5 $\frac{dy}{dx}-y=\cos x$

Q 6 $x\frac{dy}{dx}+2y=x^2$

Q 7 $\frac{dy}{dx}+2y=\sin x$

Q 8 $\frac{dy}{dx}+\frac{y}{x}=x^2$

Q 9 $\cos^2x\frac{dy}{dx}+y=\tan x$

Q 10 $(1+x^2)dy+2xy dx=\cot x dx\ ,(x\neq 0)$

Q 11 $(x+y)\frac{dy}{dx}=1$

Q 12 $x\frac{dy}{dx}=y-x\tan\frac{y}{x}$

Long Question (5 Marks)

Solve (हल करें) : -

Q 1 $(1+e^{x/y})dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$

Q 2. $\left(x\cos\frac{y}{x}+y\sin\frac{y}{x}\right)y dx=\left(y\sin\frac{y}{x}-x\cos\frac{y}{x}\right)x dy$

Q 3 $y dx+x\log\left(\frac{y}{x}\right)dy-2x dy=0$

Q 4 $(x^2-1)\frac{dy}{dx}+2xy=\frac{2}{x^2-1}$

Q 5 $(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x$

Q 6 $\frac{dy}{dx}-2y=\cos 3x$

Q 7 $(x\log x)\frac{dy}{dx}+y=\frac{2}{x}\log x$

Q 8 $\frac{dy}{dx}-3y\cot x=\sin 2x$ **, given that y=2, when** $x=\frac{\pi}{2}$

Q 9 $\left(1+y^2\right)dx = \left(\tan^{-1}y - x\right)dy$

Q 10 $\dfrac{dy}{dx} + xy = xy^3$

MCQ 1-Marks Solution

Ans : -

1 - (a)   2 - (a)   3 - (c)   4 - (b)   5 - (b)   6 - (b)   7 - (c)

8 - (a)   9 - (b)   10 - (c)

2-Marks Solution

1 Ans : -

Given curve is $x^2+y^2=2ax$ ....(1)

दिया गया वक्र है $x^2+y^2=2ax$ ....(1)

on differentiating both sides w.r. to x,

दोनों तरफ x के सापेक्ष अवकलन करने पर,

$$2x + 2y\frac{dy}{dx} = 2a \quad ....(2)$$

$\therefore$ From (1) and (2)

(1) तथा (2) से ,

$$x^2 + y^2 = \mathbf{x}\left(2x + 2y\frac{dy}{dx}\right)$$

or , $x^2 + y^2 = 2x^2 + 2xy\dfrac{dy}{dx}$

or , $2xy\dfrac{dy}{dx} + x^2 - y^2 = 0$

which is the required differential equation .

यही अभीष्ट अवकल समीकरण है।

2 Ans : -

Let P denote the family of parabolas and let (0,a) be the focus of a member of the given family , where a is an arbitrary constant.

माना कि P परवलय के कुल है तथा (0,a) नाभि का नियामक है,जहाँ a कोई स्वेच्छ अचर है।

$\therefore$ Equation of family P is

अतः वक्रों के कुल P का समीकरण है

$$x^2=4ay \quad ....(1)$$

Differentiating both sides of equation (1) w.r. to x, we get,

समीकरण (1) को दोनों तरफ x के सापेक्ष अवकलित करने पर,

$$2x = 4a\frac{dy}{dx}$$

$\therefore 2x = \dfrac{x^2}{y}\dfrac{dy}{dx}$ [from (1), $4a = \dfrac{x^2}{y}$]

or , $2y = x\dfrac{dy}{dx}$

or , $x\dfrac{dy}{dx} - 2y = 0$

which is the differential equation of the given family of parabolas.

यही दिये गए परवलयों के कुल का अवकल समीकरण है।

3 Ans : -

Given differential equation is $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$

दिया गया अवकल समीकरण है

or , $\dfrac{dy}{1+y^2} = \dfrac{dx}{1+x^2}$

Integrating both sides, we get

दोनों तरफ समाकलित करने पर,

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

or, $\tan^{-1}y = \tan^{-1}x + c$

or, $\tan^{-1}y - \tan^{-1}x = c$

or , $\tan^{-1}\dfrac{y-x}{1+yx} = c$

or , $\dfrac{y-x}{1+xy} = \tan c$

or , $\dfrac{y-x}{1+xy} = k$ (where $k = \tan c$)

which is the required general solution.

यही अभीष्ट व्यापक हल है।

4 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is $\dfrac{dy}{dx} = \sqrt{4-y^2}$

or , $\dfrac{dy}{\sqrt{4-y^2}} = dx$

on integrating both sides, we get

दोनों तरफ समाकलित करने पर

$$\int \frac{dy}{\sqrt{4-y^2}} = \int dx$$

or , $\displaystyle\int \frac{dy}{\sqrt{2^2-y^2}} = \int dx$

or , $\sin^{-1}\left(\dfrac{y}{2}\right) = x + c$

or , $\frac{y}{2} = \sin(x + c)$

$\therefore y = 2\sin(x + c)$

which is the required general solution.

यही अभीष्ट व्यापक हल है।

5 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is $\frac{dy}{dx} = \frac{x - 1}{y + 2}$

or $(y + 2)dy = (x - 1)dx$

दोनो तरफ समाकलित करने पर

integrating both sides,we get

$\int (y + 2)dy = \int (x - 1)dx$

or , $\frac{y^2}{2} + 2y = \frac{x^2}{2} - x + c$

or , $\frac{y^2 + 4y}{2} = \frac{x^2 - 2x + 2c}{2}$

$y^2 + 4y = x^2 - 2x + k$, where $k = 2c$

$y^2 + 4y - x^2 + 2x = k$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

6 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$\frac{dy}{dx} = 1 - x + y - xy$

or , $\frac{dy}{dx} = 1(1 - x) + y(1 - x)$

or , $\frac{dy}{dx} = (1 - x)(1 + y)$

or , $\frac{dy}{1 + y} = (1 - x)dx$

दोनो तरफ समाकलित करने पर

integrating both sides , we get

$\int \frac{dy}{1 + y} = \int (1 - x)dx$

or , $\log|1 + y| = x - \frac{x^2}{2} + c$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

7 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$

or , $\frac{dy}{1 + y^2} = (1 + x^2)dx$

दोनो तरफ समाकलित करने पर

integrating both sides , we get

$\int \frac{dy}{1 + y^2} = \int (1 + x^2)dx$

or , $\tan^{-1} y = x + \frac{x^3}{3} + c$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

8 Ans : -

$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$

or , $dy = \left(\frac{1 - \cos x}{1 + \cos x}\right)dx$

or , $dy = \frac{2\sin^2 x/2}{2\cos^2 x/2} dx$

or , $dy = \tan^2 x/2 \, dx$

or ,$dy = \left(\sec^2 x/2 - 1\right)dx$

दोनो तरफ समाकलित करने पर

integrating both sides , we get

$\int dy = \int \left(\sec^2 x/2 - 1\right)dx$

or , $y = \frac{\tan x/2}{\frac{1}{2}} - x + c$

or , $y = 2\tan x/2 - x + c$

Which is the required genreal solution

यही अभीष्ट व्यापक हल है।

9 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is $x\frac{dy}{dx} - y = x^2$

or $\frac{dy}{dx} - \frac{1}{x} . y = x$ ....(1)

यह $\frac{dy}{dx}+Py=Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P=-\frac{1}{x}$, तथा $Q=X$

which is a linear differential equation of the from $\frac{dy}{dx}+Py=Q$, where $P=-\frac{1}{x}$, $Q=x$

Now I.F.$=e^{\int Pdx}=e^{\int -\frac{1}{x}dx}=e^{-\log x}=e^{\log\frac{1}{x}}$

$$\Rightarrow \text{I.F} = \frac{1}{x}$$

$\therefore$ Solution of the d.e.(1) will be

अतः अवकल समीकरण (1) का हल होगा : -

$$y\cdot\frac{1}{x}=\int\frac{1}{x}\cdot x dx+c$$

or , $\frac{y}{x}=x+c \quad \Rightarrow y=x^2+cx$

यही अभीष्ट व्यापक हल है।

Which is the required general solution .

10 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is $x\frac{dy}{dx}=x+y$

or , $\frac{dy}{dx}=1+\frac{y}{x}$

or , $\frac{dy}{dx}-\frac{1}{x}.y=1$ ....(1)

which is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=-\frac{1}{x}$, $Q=1$

यह $\frac{dy}{dx}+Py=Q$ के रूप का रैखिक अवकल समीकरण है जहाँ $P=-\frac{1}{x}$, $Q=1$

Now, I.F.$=e^{\int Pdx}=e^{\int -\frac{1}{x}dx}=e^{-\log x}=e^{\log\frac{1}{x}}=\frac{1}{x}$

$\therefore$ Solution of the d.e.(1) will be

अतः अवकल समीकरण (1) का हल होगा : -

$$y.\frac{1}{x}=\int\frac{1}{x}.1dx+c$$

or , $\frac{y}{x}=\log|x|+c$

or , $y=x\log|x|+cx$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

3-Marks Solution

1 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$$\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$$

dividing by tanx.tany , we get

$$\frac{\sec^2 x}{\tan x}dx+\frac{\sec^2 y}{\tan y}dy=0$$

समाकलित करने पर

Integrating , we get

$$\int\frac{\sec^2 x}{\tan x}dx+\int\frac{\sec^2 y}{\tan y}dy=\log c$$

or , $\log|\tan x|+\log|\tan y|=\log c$

or , $\log|\tan x.\tan y|=\log c$

or , $|\tan x.\tan y|=c$

$\therefore \tan x.\tan y=\pm c$

or , $\tan x.\tan y=k \quad$ where $k=\pm c$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

2 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is $y\log y dx-x dy=0$

or , $x dy=y\log y dx$

or , $\frac{dy}{y\log y}=\frac{dx}{x}$ ....(1)

दोनों तरफ समाकलित करने पर

Integrating both sides , we get

$$\int\frac{dy}{y\log y}=\int\frac{dx}{x}$$

or , $\log(\log y)=\log|x|+\log|c|$

$\Rightarrow \log(\log y)=\log|cx|$

$\Rightarrow \log y=cx \quad \Rightarrow y=e^{cx}$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

3 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$$e^x\tan y dx+(1-e^x)\sec^2 y dy=0$$

or , $\frac{e^x}{1-e^x}dx+\frac{\sec^2 y}{\tan y}dy=0$

समाकलित करने पर

Integrating , we get

$$\int \frac{e^x}{1-e^x}dx + \int \frac{\sec^2 y}{\tan y}dy = \log c$$

or , $-\log(1-e^x) + \log\tan y = \log c$

or , $\log\frac{\tan y}{1-e^x} = \log c$

or , $\frac{\tan y}{1-e^x} = c \quad \Rightarrow \tan y = c(1-e^x)$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

4 Ans ; -

दिया गया अवकल समीकरण है

Given differential equation is

$$xdy - ydx = \sqrt{x^2+y^2}\,dx$$

or , $\quad xdy = \left(y+\sqrt{x^2+y^2}\right)dx$

or , $\quad \frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$ ....(1)

Which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

put $y = vx \qquad \therefore \; \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ Equation (1) becomes

$\therefore$ समीकरण(1) हो जाता है

$$v + x\frac{dv}{dx} = \frac{vx+\sqrt{x^2+v^2x^2}}{x}$$

or , $x\frac{dv}{dx} = \sqrt{1+v^2} \quad$ or , $\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$

समाकलित करने पर

Integrating , we get

$$\log\left|v+\sqrt{1+v^2}\right| = \log|x| + \log k$$

$\Rightarrow \quad \left|v+\sqrt{1+v^2}\right| = k|x|$

$\Rightarrow \quad v+\sqrt{1+v^2} = \pm kx = cx$, where $c = \pm k$

$\therefore \quad \frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}} = cx$

$\Rightarrow \quad y + \sqrt{x^2+y^2} = cx^2$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

5 Ans ; -

दिया गया अवकल समीकरण है

Given differential equation is $\frac{dy}{dx} - y = \cos x$ ...(1)

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $\quad P = -1, \quad Q = \cos x$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल समीकरण है जहाँ $P = -1, Q = \cos x$

$\therefore \;$ I.F $= e^{\int p dx} = e^{\int -1 dx} = e^{-x}$

$\therefore$ Solution of d.e. (1) , will be

अतः अवकल समीकरण (1) का हल होगा : -

$$y.e^{-x} = \int e^{-x}.\cos x dx + c \;\; ....(2)$$

Let $I = \int e^{-x}\cos x dx$

$= \cos x.(-e^{-x}) - \int -e^{-x}(-\sin x)dx$

$= -e^{-x}\cos x - \int e^{-x}\sin x dx$

$= -e^{-x}\cos x - \left[\sin x(-e^{-x}) - \int \cos x(-e^{-x})dx\right]$

$= -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x dx$

or , $I = e^{-x}(\sin x - \cos x) - I$

or , $2I = e^{-x}(\sin x - \cos x)$

$\therefore I = \frac{e^{-x}(\sin x - \cos x)}{2}$

$\therefore$ From eqn (2) , $y.e^{-x} = \frac{e^{-x}(\sin x - \cos x)}{2} + c$

$\therefore \;\; y = \frac{\sin x - \cos x}{2} + ce^x$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

6 Ans : -Given differential equation is

दिया गया अवकल समीकरण है

$$x\frac{dy}{dx} + 2y = x^2 \quad \text{or , } \frac{dy}{dx} + \frac{2}{x}y = x \;\; ......(1)$$

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q \quad$ where $P = \frac{2}{x}, \quad Q = x$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P = \frac{2}{x}, Q = x$

$\therefore \;$ I.F. $= e^{\int P dx} = e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$

$\therefore \;$ Solution of d.e. (1) will be

$\therefore$ अवकल समीकरण (1) का हल होगा

$$y.x^2 = \int x^2.x dx + c$$

or , $y.x^2 = \frac{x^4}{4} + c$

$\therefore\ y = \frac{x^2}{4} + \frac{c}{x^2}$

Which is the general solution of given d.e.

यही अभीष्ट व्यापक हल है।

7 Ans : -

दिया गया अवकल समीकरण है

Given d.e. is $\frac{dy}{dx} + 2y = \sin x$ ....(1)

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = 2$, $Q = \sin x$

यह $\frac{dy}{dx} + Py = Q$ **के रूप का रैखिक अवकल समीकरण है, जहाँ** $P = 2$, $Q = \sin x$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int 2 dx} = e^{2x}$

$\therefore$ Solution of d.e. (1) will be

$\therefore$ **अवकल समीकरण (1) का हल होगा**

$y.e^{2x} = \int e^{2x}.\sin x dx + c$ ....(2)

Let $I = \int e^{2x} \sin x dx$

$= \sin x.\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} \cdot \cos x dx$

$= \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2}\left[\cos x\left(\frac{e^{2x}}{2}\right) - \int \frac{e^{2x}}{2} \cdot (-\sin x) dx\right]$

$= \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cdot \cos x}{4} - \frac{1}{4}\int e^{2x} \sin x$

or , $I = \frac{e^{2x}(2\sin x - \cos x)}{4} - \frac{1}{4} I$

or , $I + \frac{1}{4} I = \frac{e^{2x}(2\sin x - \cos x)}{4}$

or , $\frac{5}{4} I = \frac{e^{2x}(2\sin x - \cos x)}{4}$

$\therefore I = \frac{e^{2x}(2\sin x - \cos x)}{5}$

$\therefore$ From eqn(2)

$\therefore$ **समी.(2)से,**

$y \cdot e^{2x} = \frac{e^{2x}(2\sin x - \cos x)}{5} + c$

$\therefore\ y = \frac{1}{5}(2\sin x - \cos x) + ce^{-2x}$

which is the required general solution.

यही अभीष्ट व्यापक हल है।

8 Ans : -

दिया गया अवकल समीकरण है

Given d.e. is $\frac{dy}{dx} + \frac{y}{x} = x^2$ ....(1)

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x}$, $Q = x^2$

यह $\frac{dy}{dx} + Py = Q$ **के रूप का रैखिक अवकल समीकरण है जहाँ** $P = \frac{1}{x}$, $Q = x^2$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा : -

$y \cdot x = \int x \cdot x^2 dx + c$

or , $xy = \int x^3 dx + c$

or , $xy = \frac{x^4}{4} + c$

or , $y = \frac{x^3}{4} + \frac{c}{x}$

which is the required general solution .

यही अभीष्ट व्यापक हल है।

9 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$\cos^2 x \frac{dy}{dx} + y = \tan x$

or , $\frac{dy}{dx} + \sec^2 x.y = \sec^2 x.\tan x$ ....(1)

which is a linear d.e. of the form $\frac{dy}{dx} + Py = Q$

where $P = \sec^2 x$, $Q = \sec^2 x.\tan x$

यह $\frac{dy}{dx} + Py = Q$ **के रूप का रैखिक अवकल समीकरण है, जहाँ** $P = \sec^2 x$, $Q = \sec^2 x \tan x$

Now , I.F. $= e^{\int P dx} = e^{\int \sec^2 x dx} = e^{\tan x}$

$\therefore$ Solution of eqn(1) will be

अतः समीकरण (1) का हल होगा : -

$y \cdot e^{\tan x} = \int e^{\tan x} \cdot \sec^2 x \cdot \tan x dx + c$

put $\tan x = t$ $\quad \therefore \sec^2 x dx = dt$

$\therefore\ y.e^{\tan x} = \int t \cdot e^t dt + c$

$= t\int e^t dt - \int\left(\frac{d(t)}{dt}\int e^t dt\right)dt + c$

$= te^t - \int e^t dt + c$

$= te^t - e^t + c$

$= \tan x . e^{\tan x} - e^{\tan x} + c$

$= e^{\tan x}(\tan x - 1) + c$

$\therefore y = (\tan x - 1) + ce^{-\tan x}$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

10 Ans : -

दिया गया अवकल समीकरण है:–

Given differential equation is

$(1+x^2)dy + 2xydx = \cot x dx$

or , $(1+x^2)\frac{dy}{dx} + 2xy = \cot x$

or , $\frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{\cot x}{1+x^2}$ ....(1)

which is a linear d.e. of the form $\frac{dy}{dx} + Py = Q$

where $P = \frac{2x}{1+x^2}, Q = \frac{\cot x}{1+x^2}$

यह $\frac{dy}{dx} + Py = Q$ **के रूप का रैखिक अवकल समीकरण है जहाँ** $P = \frac{2x}{1+x^2}, Q = \frac{\cot x}{1+x^2}$

Now , I.F. $= e^{\int Pdx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1 + x^2$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा : -

$y(1+x^2) = \int (1+x^2) \cdot \frac{\cot x}{1+x^2} dx + c$

or , $y(1+x^2) = \int \cot x dx + c$

or , $y(1+x^2) = \log|\sin x| + c$

$\therefore \quad y = (1+x^2)^{-1}\log|\sin x| + c(1+x^2)^{-1}$

Which is required general solution.

यही अभीष्ट व्याप्क हल है।

11 Ans : -

$(x+y)\frac{dy}{dx} = 1$, or , $\frac{dx}{dy} = x + y$

or , $\frac{dx}{dy} - x = y$ ....(1)

which is a linear d.e. of the form

$\frac{dx}{dy} + Px = Q$, where , $P = -1, Q = y$

यह $\frac{dx}{dy} + Px = Q$ **के रूप का रैखिक अवकल समीकरण है जहाँ** $P = -1, Q = y$

$\therefore$ I.F. $= e^{\int Pdy} = e^{\int -1dy} = e^{-y}$

$\therefore$ Solution of eqn(1) will be

अतः समीकरण (1) का हल होगा : -

$x \cdot e^{-y} = \int ye^{-y}dy + c$

or , $x \cdot e^{-y} = -ye^{-y} - \int -e^{-y}dy + c$

$= -ye^{-y} - e^{-y} + c \quad = -e^{-y}(1+y) + c$

$\therefore \quad x = -(1+y) + ce^{y}$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

12 Ans : - Given d.e. is $x\frac{dy}{dx} = y - x\tan\frac{y}{x}$

दिया गया अवकल समीकरण है

$\frac{dy}{dx} = \frac{y}{x} - \tan\frac{y}{x}$ ....(1)

which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

$\therefore$ Put y=vx

$\therefore \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ Equation (1) becomes

समीकरण (1) हो जाता है

$v + x\frac{dv}{dx} = v - \tan v$

$\Rightarrow x\frac{dv}{dx} = -\tan v$

or , $\frac{dv}{\tan v} = -\frac{dx}{x}$

$\Rightarrow \int \frac{dv}{\tan v} = -\int \frac{dx}{x}$

or , $\int \cot v dv = -\log|x| + \log c$

or , $\log(\sin v) + \log|x| = \log c$

$\Rightarrow \log|x\sin v| = \log c$

$\Rightarrow |x\sin v| = c$

$\Rightarrow x\sin v = \pm c = k$ (say)

$\Rightarrow x\sin\frac{y}{x} = k$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

5-Marks Solution

1 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$$(1+e^{x/y})dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$$

or , $\frac{dx}{dy} = -\frac{e^{x/y}\left(1-\frac{x}{y}\right)}{1+e^{x/y}}$ ....(1)

which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

Put $x = vy$, then $\frac{dx}{dy} = v + y\frac{dv}{dy}$

$\therefore$ From eqn(1) ,

अतः समीकरण (1) से : -

$$v + y\frac{dv}{dy} = -\frac{e^v(1-v)}{1+e^v}$$

$$\therefore\ y\frac{dv}{dy} = -\frac{e^v(1-v)}{1+e^v} - v$$

$$= \frac{-e^v(1-v) - v(1+e^v)}{1+e^v}$$

or , $y\frac{dv}{dy} = -\frac{v+e^v}{1+e^v}$

or , $\frac{1+e^v}{v+e^v}dv = -\frac{dy}{y}$

$\Rightarrow \int \frac{1+e^v}{v+e^v}dv = -\int \frac{dy}{y}$

$\Rightarrow \log|v+e^v| = -\log|y| + c$

$\Rightarrow \log|y(v+e^v)| = c$

$\Rightarrow |y(v+e^v)| = e^c$

$\Rightarrow y\left(\frac{x}{y} + e^{x/y}\right) = \pm e^c = k(\text{say})$

$\therefore\quad x + ye^{x/y} = k$

Which is the required general solution .

यही अभीष्ट व्यापक हल है।

2 Ans : -

दिया गया अवकल समीकरण है–

Given differential equation is

$$y\left(x\cos\frac{y}{x} + y\sin\frac{y}{x}\right)dx = x\left(y\sin\frac{y}{x} - x\cos\frac{y}{x}\right)dy$$

or , $y\left(x\cos\frac{y}{x} + y\sin\frac{y}{x}\right) = x\left(y\sin\frac{y}{x} - x\cos\frac{y}{x}\right)\frac{dy}{dx}$

or , $\frac{dy}{dx} = \frac{\left\{x\cos\frac{y}{x} + y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\frac{y}{x} - x\cos\frac{y}{x}\right\}x}$

dividing num. and den. by x^2 , we get

अंश तथा हर को x^2 से विभाजित करने पर,

$$\frac{dy}{dx} = \frac{\left\{\cos\frac{y}{x} + \frac{y}{x}\sin\left(\frac{y}{x}\right)\right\}\frac{y}{x}}{\left\{\frac{y}{x}\sin\frac{y}{x} - \cos\frac{y}{x}\right\}} \quad ....(1)$$

Put $y = vx$, then $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ From (1) , $v + x\frac{dv}{dx} = \frac{(\cos v + v\sin v)v}{v\sin v - \cos v}$

or , $x\frac{dv}{dx} = \frac{v(\cos v + v\sin v)}{v\sin v - \cos v} - v$

or , $x\frac{dv}{dx} = \frac{v(\cos v + v\sin v - v\sin v + \cos v)}{v\sin v - \cos v}$

$= \frac{2v\cos v}{v\sin v - \cos v}$

or , $\frac{v\sin v - \cos v}{v\cos v}dv = \frac{2dx}{x}$

or , $\int \frac{v\sin v - \cos v}{v\cos v}dv = 2\int \frac{1}{x}dx$

$\Rightarrow \int \left(\tan v - \frac{1}{v}\right)dv = 2\log|x| + \log c$

or , $-\log|\cos v| - \log|v| = 2\log|x| + \log c$

or , $-\log|v\cos v| = 2\log|x| + \log c$

or , $2\log|x| + \log|v\cos v| = -\log c$

or , $\log|x^2 v\cos v| = \log k \quad (-\log c = \log k)$

$\Rightarrow |x^2 v\cos v| = k \Rightarrow x^2 v\cos v = \pm k = c$

(Where $\pm k = c$)

$\Rightarrow x^2\frac{y}{x}\cos\frac{y}{x} = c \quad \Rightarrow xy\cos\frac{y}{x} = c$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

3 Ans : -दिया गया अवकल समीकरण है

Given differential equation is

$y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0$ ....(1)

or , $y\frac{dx}{dy} = 2x - x\log\left(\frac{y}{x}\right)$

or , $\frac{dx}{dy} = 2\frac{x}{y} - \frac{x}{y}\log\left(\frac{y}{x}\right)$

or , $\frac{dx}{dy} = 2\cdot\frac{x}{y} + \frac{x}{y}\log\left(\frac{x}{y}\right)$ ....(2)

$$\left(\because \log a = -\log\frac{1}{a}\right)$$

which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

Put $x = vy$, then $\frac{dx}{dy} = v + y\frac{dv}{dy}$

$\therefore$ From eqn(2) , $v + y\frac{dv}{dy} = 2v + v\log v$

$$\Rightarrow y\frac{dv}{dy} = v + v\log v$$

$$\Rightarrow \frac{dv}{v(1+\log v)} = \frac{dy}{y}$$

$$\Rightarrow \int\frac{dv}{v(1+\log v)} = \int\frac{dy}{y}$$

$$\Rightarrow \log|1+\log v| = \log|y| + c$$

$$\Rightarrow \log\left|\frac{1+\log v}{y}\right| = c$$

$$\Rightarrow \left|\frac{1+\log v}{y}\right| = e^c$$

$$\therefore \quad \frac{1+\log v}{y} = \pm e^c = k(\text{say})$$

$$\Rightarrow 1 + \log\frac{x}{y} = ky ,$$

Where k is an arbitrary constant.

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

4 Ans : -

दिया गया अवकल समीकरण है

Given d.e. is $(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$

or , $\frac{dy}{dx} + \frac{2x}{x^2-1}\cdot y = \frac{2}{(x^2-1)^2}$ ....(1)

which is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

where $P = \frac{2x}{x^2-1}$, $Q = \frac{2}{(x^2-1)^2}$

यह $\frac{dy}{dx} + Py = Q$ **के रूप का रैखिक अवकल समीकरण है, जहाँ** $P = \frac{2x}{x^2-1}$, $Q = \frac{2}{(x^2-1)^2}$

$$\therefore \text{ I.F.} = e^{\int P dx} = e^{\int\frac{2x}{x^2-1}dx} = e^{\log(x^2-1)} = x^2 - 1$$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा : -

$$y(x^2-1) = \int (x^2-1)\cdot\frac{2}{(x^2-1)^2}\cdot dx + c$$

$$= \int\frac{2}{x^2-1}dx + c$$

$$\therefore \; y(x^2-1) = 2\cdot\frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + c$$

$$\text{or , } y(x^2-1) = \log\left|\frac{x-1}{x+1}\right| + c$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

5 Ans ; -

दिया गया अवकल समीकरण है

Given differential equation is

$$(1+x^2)\frac{dy}{dx} + y = \tan^{-1}x$$

or , $\frac{dy}{dx} + \frac{1}{1+x^2}\cdot y = \frac{\tan^{-1}x}{1+x^2}$ ....(1)

which is a linear d.e. of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{1+x^2}$, $Q = \frac{\tan^{-1}x}{1+x^2}$

यह $\frac{dy}{dx} + Py = Q$ **के रूप का रैखिक अवकल समीकरण है, जहाँ** $P = \frac{1}{1+x^2}$, $Q = \frac{\tan^{-1}x}{1+x^2}$

Now , $\text{I.F.} = e^{\int P dx} = e^{\int\frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा : -

$$y.e^{\tan^{-1}x} = \int e^{\tan^{-1}x}\cdot\frac{\tan^{-1}x}{1+x^2}dx + c$$

Put $\tan^{-1}x = t$ $\quad\therefore \frac{1}{1+x^2}dx = dt$

$$\therefore \quad y.e^{\tan^{-1}x} = \int te^t dt + c$$

$$= t\int e^t dt - \int\left\{\frac{d(t)}{dt}\int e^t dt\right\}dt + c$$

$$= te^t - \int e^t dt + c$$

$$= te^t - e^t + c$$

$$\therefore \; y\cdot e^{\tan^{-1}x} = (t-1)e^t + c$$

$$\text{or , } y\cdot e^{\tan^{-1}x} = (\tan^{-1}x - 1)e^{\tan^{-1}x} + c$$

$$\therefore \; y = \tan^{-1}x - 1 + ce^{-\tan^{-1}x}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

6 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$\frac{dy}{dx} - 2y = \cos 3x$ ....(1)

which is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=-2$, $Q=\cos 3x$

यह $\frac{dy}{dx}+Py=Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P=-2$, $Q=\cos 3x$

Now, I.F. $= e^{\int P dx} = e^{\int -2dx} = e^{-2x}$

$\therefore$ Solution of the equation (1) will be

अतः समीकरण (1) का हल होगा :-

$$y \cdot e^{-2x} = \int e^{-2x}\cos 3x\, dx + c \quad ....(2)$$

Let $I = \int e^{-2x}\cos 3x\, dx$

then

$$I = \cos 3x\int e^{-2x}dx - \int\left\{\frac{d}{dx}(\cos 3x)\int e^{-2x}dx\right\}dx$$

$$= \frac{-e^{-2x}\cos 3x}{2} - \int(-3\sin 3x)\left(\frac{-e^{-2x}}{2}\right)dx$$

$$= \frac{-e^{-2x}\cos 3x}{2} - \frac{3}{2}\int e^{-2x}\sin 3x\, dx$$

$$= \frac{-e^{-2x}\cos 3x}{2} - \frac{3}{2}\left[-\sin 3x\cdot\frac{e^{-2x}}{2} - \int 3\cos 3x\cdot\frac{e^{-2x}}{-2}dx\right]$$

$$= \frac{-e^{-2x}\cos 3x}{2} + \frac{3e^{-2x}\sin 3x}{4} - \frac{9}{4}\int e^{-2x}\cos 3x\, dx$$

or, $I = \frac{-2e^{-2x}\cos 3x + 3e^{-2x}\sin 3x}{4} - \frac{9}{4}I$

or, $I + \frac{9}{4}I = \frac{e^{-2x}(-2\cos 3x + 3\sin 3x)}{4}$

or, $\frac{13}{4}I = \frac{e^{-2x}(-2\cos 3x + 3\sin 3x)}{4}$

$\Rightarrow I = \frac{1}{13}e^{-2x}(-2\cos 3x + 3\sin 3x)$

$\therefore$ From eqn(2),

$$y\cdot e^{-2x} = \frac{e^{-2x}}{13}(3\sin 3x - 2\cos 3x) + c$$

$$\Rightarrow y = \frac{1}{13}(3\sin 3x - 2\cos 3x) + ce^{2x}$$

Which is the required general solution.
यही अभीष्ट व्यापक हल है।

7 Ans :-

दिया गया अवकल समीकरण है

Given d.e. is $x\log x\cdot\frac{dy}{dx} + y = \frac{2}{x}\log x$

or, $\frac{dy}{dx} + \frac{1}{x\log x}\cdot y = \frac{2}{x^2}$ ....(1)

which is a linear d.e.of the form $\frac{dy}{dx}+Py=Q$

where $P = \frac{1}{x\log x}$ and $Q = \frac{2}{x^2}$

यह $\frac{dy}{dx}+Py=Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P = \frac{1}{x\log x}$ and $Q = \frac{2}{x^2}$

Now, I.F. $= e^{\int P dx} = e^{\int\frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा :-

$$y\cdot\log x = \int\frac{2}{x^2}\cdot\log x\, dx + c$$

$$= 2\left[\log x\int\frac{1}{x^2}dx - \int\left\{\frac{d(\log x)}{dx}\int\frac{1}{x^2}dx\right\}dx\right] + C$$

$$= 2\left[\log x\left(-\frac{1}{x}\right) - \int\frac{1}{x}\left(-\frac{1}{x}\right)dx\right] + c$$

$$= -\frac{2\log x}{x} + 2\int\frac{1}{x^2}dx + c$$

$$= -\frac{2\log x}{x} - \frac{2}{x} + c$$

$$= -\frac{2}{x}(\log x + 1) + c$$

$$\therefore\ y\cdot\log x = -\frac{2}{x}(\log x + 1) + c$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

8 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$\frac{dy}{dx} - 3\cot x.y = \sin 2x \quad ....(1)$$

which is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=-3\cot x$, $Q=\sin 2x$

यह $\frac{dy}{dx}+Py=Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P=-3\cot x$ and $Q=\sin 2x$

Now, I.F. $= e^{\int P dx} = e^{\int -3\cot x dx} = e^{-3\log\sin x} = e^{\log(\sin x)^{-3}}$

$$= (\sin x)^{-3} = \frac{1}{\sin^3 x}$$

$\therefore$ Solution of the given differential equation(1) will be

अतः दिए गए अवकल समीकरण (1) का हल होगा :-

$$y\times\frac{1}{\sin^3 x} = \int\left(\sin 2x\times\frac{1}{\sin^3 x}\right)dx + c$$

or, $\frac{y}{\sin^3 x} = \int\frac{2\sin x\cdot\cos x}{\sin^3 x}dx + c$

$$= 2\int\frac{\cos x}{\sin^2 x}dx + c$$

$$= 2\int \frac{dt}{t^2} + c \quad \text{where } \sin x = t$$

$$= -\frac{2}{t} + c = -\frac{2}{\sin x} + c$$

$$\therefore \frac{y}{\sin^3 x} = -\frac{2}{\sin x} + c$$

$$\therefore y = -2\sin^2 x + c\sin^3 x \quad \text{.....................(2)}$$

Now , given $y = 2$ when $x = \frac{\pi}{2}$

$\therefore$ From eqn(2) , $2 = -2 + c \Rightarrow c = 4$

$$\therefore \quad y = -2\sin^2 x + 4\sin^3 x$$

Which is the required particular solution.

यही अभीष्ट विशेष हल है।

9 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$$(1 + y^2)dx = (\tan^{-1} y - x)dy$$

$$\text{or , } \frac{dx}{dy} = \frac{\tan^{-1} y - x}{1 + y^2}$$

$$\text{or , } \frac{dx}{dy} = \frac{\tan^{-1} y}{1 + y^2} - \frac{1}{1 + y^2} \cdot x$$

$$\text{or , } \frac{dx}{dy} + \frac{1}{1 + y^2} \cdot x = \frac{\tan^{-1} y}{1 + y^2} \quad \text{..........(1)}$$

which is a linear diff. equation of the form

$\frac{dx}{dy} + Px = Q$, where $P = \frac{1}{1 + y^2}$, $Q = \frac{\tan^{-1} y}{1 + y^2}$

यह $\frac{dx}{dy} + Px = Q$ **के रूप का रैखिक अवकल समीकरण है, जहाँ** $P = \frac{1}{1 + y^2}$, $Q = \frac{\tan^{-1} y}{1 + y^2}$

$$\therefore \quad \text{I.F.} = e^{\int P dy} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

$\therefore$ Solution of the differential equation (1) will be

अतः अवकल समीकरण (1) **का हल होगा : -**

$$x \cdot e^{\tan^{-1} y} = \int e^{\tan^{-1} y} \cdot \frac{\tan^{-1} y}{1 + y^2} dy + c$$

$$= \int te^t dt + c \ , \quad \text{where} \quad \tan^{-1} y = t$$

$$= t\int e^t dt - \int \left\{ \frac{d(t)}{dt} . \int e^t dt \right\} dt + c$$

$$= te^t - \int e^t dt + c$$

$$= te^t - e^t + c$$

$$= e^t (t - 1) + c$$

$$\therefore \quad x \cdot e^{\tan^{-1} y} = e^{\tan^{-1} y} (\tan^{-1} y - 1) + c$$

$$\therefore \quad x = \tan^{-1} y - 1 + ce^{-\tan^{-1} y}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

10 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$$\frac{dy}{dx} + xy = xy^3 \quad \text{.............(1)}$$

$$\text{or , } \frac{1}{y^3} \cdot \frac{dy}{dx} + \frac{xy}{y^3} = x$$

$$\text{or , } \frac{1}{y^3} \frac{dy}{dx} + \frac{1}{y^2} \cdot x = x \quad \text{..................(2)}$$

$$\text{Put } \frac{1}{y^2} = t \qquad \therefore -\frac{2}{y^3} \frac{dy}{dx} = \frac{dt}{dx}$$

$$\text{or , } \frac{1}{y^3} \frac{dy}{dx} = -\frac{1}{2} \frac{dt}{dx} \quad \text{.....................(3)}$$

$\therefore$ Equation (2) becomes

$$-\frac{1}{2} \frac{dt}{dx} + t.x = x$$

$$\text{or , } \frac{dt}{dx} - 2x \cdot t = -2x \quad \text{............(4)}$$

which is a linear differential equation of the form

$\frac{dt}{dx} + Pt = Q$, where $P = -2x$ and $Q = -2x$

यह $\frac{dt}{dx} + Pt = Q$ **के रूप का रैखिक अवकल समीकरण है जहाँ** $P = -2x$ and $Q = -2x$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{\int -2x dx} = e^{-x^2}$$

$\therefore$ Solution of equation (4) will be

अतः समीकरण (4) **का हल होगा : -**

$$t.e^{-x^2} = \int e^{-x^2} (-2x) dx + c$$

$$= e^{-x^2} + c$$

$$\Rightarrow \frac{1}{y^2} \cdot e^{-x^2} = e^{-x^2} + c$$

$$\Rightarrow \frac{1}{y^2} = 1 + ce^{x^2}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।