# Chapter - 4

# Determinants
# सारणिक

**Multiple choice Questions**

**बहुविकल्पीय प्रश्न**

**Q.1. IF $A = \begin{bmatrix} 2 & 4 \\ -5 & -1 \end{bmatrix}$ then $|A|$ ?**

यदि $\begin{bmatrix} 2 & 4 \\ -5 & -1 \end{bmatrix}$ तो $|A| = ?$

(a) -18 (b) 18

(c) 0 (d) 12

**Q.2. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix}$ then x is equal to**

यदि $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix}$ तो x का मान है –

(a) 6 (b) $\pm 6$

(c) -6 (d) zero

**Q.3. $\begin{vmatrix} \sin 10^o & -\cos 10^o \\ \sin 80^o & \cos 80^o \end{vmatrix} =$**

(a) 1 (b) -1

(c) $\pm 1$ (d) 0

**Q.4. If A is a square Matrix then $|A'| = ?$**

यदि A एक वर्ग आव्यूह है तो $|A'| = ?$

(a) A (b) $-|A|$

(c) $|A|$ (d) A'

**Q.5. Let A be a square Matrix of order $3 \times 3$ and $|A| = 2$ then $|2A|$ is euqal to**

माना A एक वर्ग आव्यूह है $3 \times 3$ कोटि का और $|A| = 2$ तो $|2A|$ का मान होगा ?

(a) 16 (b) -16

(c) 8 (d) -8

**Q.6. If I is an Identity Matrix then $|I| = ?$**

यदि I एक तत्समक आव्यूह है तो $|I| = ?$

(a) -1 (b) -2

(c) 1 (d) 2

**Q.7. If $\begin{vmatrix} x & 2 \\ 2 & 1 \end{vmatrix} = 0$ then value of x is**

यदि $\begin{vmatrix} x & 2 \\ 2 & 1 \end{vmatrix} = 0$ तो x का मान है

(a) 2 (b) -4

(c) 5 (d) 1

**Q.8. $\begin{vmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 2 \end{vmatrix} = ?$**

(a) 20 (b) 30

(c) 40 (d) -20

**Q.9. Determinant of null square matrix will be**

शुन्य वर्ग आव्यूह का सारणिक मान होगा?

(a) 0 (b) 1

(c) 6 (d) None

**Q.10. Area of triangle is always**

त्रिभुज का क्षेत्रफल हमेशा होता है –

(a) Negative ऋणात्मक (b) Positive धनात्मक

(c) Zero शून्य (d) None of these इनमें से कोई नहीं

**Q.11. A square Matrix A is invertible if**

एक वर्ग आव्यूह व्युत्क्रमणीय है, यदि –

(a) $|A| = 0$ (b) $|A| = 1$

(c) $|A| \neq 0$ (d) None of these

**Q.12. If A is invertible matrix then $A^{-1}$ will be**

यदि A व्युत्क्रमणीय आव्यूह है तो $A^{-1}$ का मान होगा

(a) adj A (b) $\frac{1}{|A|}$ adj A

(c) $\frac{-1}{|A|}$adj A (d) None of these

**Q.13. $\begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 8 \end{vmatrix} = ?$**

(a) 8 (b) 40

(c) 48 (d) 50

**Q.14.** $\begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix} = ?$

(a) - 4 (b) 2

(c) 6 (d) 0

**Q.15.** $\begin{vmatrix} 0 & 0 \\ 5 & 8 \end{vmatrix} = ?$

(a) 5 (b) 40

(c) 8 (d) 0

**Q.16. Which of the following is correct?**

**निम्नलिखित में से कौन सही है?**

(a) $|A| = |A'|$ (b) $|A| = -|A'|$

(c) $|A| \neq |A'|$ (d) None

**Q.17. Value of the determinant of a matrix exist , if matrix is -**

**किसी आव्यूह के सारणिक मान संभव है यदि आव्यूह है :–**

(a) square matrix (वर्ग आव्यूह) (b) Rectangle Matrix (आयत आव्यूह)

(c) Any Matrix कोई भी आव्यूह (d) None of these इनमें से कोई नहीं

**Q.18. If** $\begin{vmatrix} x & 2 \\ 3 & 1 \end{vmatrix} = 8$ **then value of x will be**

**यदि** $\begin{vmatrix} x & 2 \\ 3 & 1 \end{vmatrix} = 8$ **तो x का मान होगा**

(a) 8 (b) 6

(c) 14 (d) 0

**Q.19.** $\begin{vmatrix} 0 & 2 & 4 \\ 0 & 0 & -7 \\ 0 & 0 & 0 \end{vmatrix} = ?$

(a) 0 (b) 8

(c) -14 (d) -56

**Q.20.** $\begin{vmatrix} 2 & 3 & 1 \\ 5 & 6 & 8 \\ 2 & 3 & 1 \end{vmatrix} = ?$

(a) 5 (b) -1

(c) 0 (d) None of these

**Q.21.** $\begin{vmatrix} 7 & 8 & -1 \\ 5 & 4 & 3 \\ 5 & 4 & 3 \end{vmatrix} = ?$

(a) 3 (b) 1

(c) 0 (d) 2

**Q.22.** $\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = ?$

(a) 0 (b) 1

(c) 2 (d) 3

**Q.23.** $\begin{vmatrix} 5 & 4 \\ 2 & 8 \end{vmatrix} = ?$

(a) 40 (b) 48

(c) 42 (d) 32

**Q.24.** $\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = ?$

(a) 2 (b) 1

(c) 4 (d) 0

**Q.25. If** $\triangle = \begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$ **then co-factor of 5 is**

**यदि** $\triangle = \begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$ **तो 5 का सहखण्ड है**

(a) 6 (b) -6

(c) 2 (d) 4

**Q.26. Minor of $a_{21}$ in** $\begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix}$ **is**

$\begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix}$ **में $a_{21}$ का Minor है**

(a) 5 (b) 2

(c) 6 (d) 0

**Q 27. If** $\begin{vmatrix} 3 & 4 & 7 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix}$ **then** $M_{11} = ?$

(a) 3 (b) 1

(c) 2 (d) -1

**Q 28. If** $A = \begin{bmatrix} 2 & 3 \\ -5 & 4 \end{bmatrix}$ **then adj A = ?**

(a) $\begin{bmatrix} 2 & 3 \\ -5 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 4 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & 4 \\ 2 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -3 \\ 5 & 2 \end{bmatrix}$

**Q 29. If $A_{n \times n}$ Matrix then $|kA| = ?$ , k is constant-**

**यदि $A_{n \times n}$ एक आव्यूह है तो $|kA| = ?$ , k नियत है।**

(a) $k|A|$ (b) $k^n|A|$

(c) $k.|A|^n$ (d) $k^n|A|^n$

**Q 30. Determinant of diagonal Matrix diag. [5, 6, 7] is-**

**विकर्ण आव्यूह diag [5, 6, 7] का सारणिक मान है:**

(a) 30 (b) 42

(c) 35 (d) 210

| MCQ Ans:- |
|---|
| 1-b, 2-b, 3-a, 4-c, 5-a, 6-c, 7-b, 8-a, 9-a, 10-b, 11-c, 12-b, 13-c, 14-a, 15-d, 16-a, 17-a, 18-c, 19-a, 20-c, 21-c, 22-a, 23-d, 24-d, 25-a, 26-c, 27-d, 28-d, 29-b, 30-d |

## Very Short Questions (2 Marks)
## अतिलघुत्तरीय प्रश्न

**Q.1. Evaluate**

मान ज्ञात करें।

(a) $\begin{vmatrix} 5 & 4 \\ 2 & 6 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \\ 5 & 0 & 3 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & 4 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 3 \end{vmatrix}$ (d) $\begin{vmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{vmatrix}$

(e) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{vmatrix}$

**Q.2. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, then find the value of x.**

यदि $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$ तो **x** का मान ज्ञात करें।

**Q.3. Write the co-factor of all elements of $\begin{vmatrix} 3 & -2 \\ 4 & 6 \end{vmatrix}$**

सारणिक $\begin{vmatrix} 3 & -2 \\ 4 & 6 \end{vmatrix}$ के सभी अवयवों के सहखण्ड लिखें।

**Q.4. Prove that $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} = 0$**

**Where, $1 + w + w^2 = 0$**

**सिद्ध करें $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} = 0$**

**जहाँ, $1 + w + w^2 = 0$**

**Q.5. If $\begin{vmatrix} x-9 & 1 \\ 7 & 1 \end{vmatrix} = 0$ , then find the value of x.**

**यदि $\begin{vmatrix} x-9 & 1 \\ 7 & 1 \end{vmatrix} = 0$ तो x का मान ज्ञात करें।**

**Q.6. Write the Matrix form of the equations**

**$2x - y = -2, 3x + 4y = 3.$**

**समीकरण $2x - y = -2, 3x + 4y = 3$ को आव्यूह रूप में लिखें।**

**Q.7. Evaluate $\begin{vmatrix} 5 & 60 & 7 \\ 3 & 36 & 2 \\ 2 & 24 & 4 \end{vmatrix}$**

## Solutions of Very Short Questions (2 Marks)

1. (a) $\begin{vmatrix} 5 & 4 \\ 2 & 6 \end{vmatrix} = 5 \times 6 - 2 \times 4$

$= 30 - 8 = 22$ Ans

(b) $\begin{vmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \\ 5 & 0 & 3 \end{vmatrix}$

Expand along $C_2$

$C_2$ के अनुदिश विस्तार

$= -3\begin{vmatrix} 1 & 2 \\ 5 & 3 \end{vmatrix} + 0\begin{vmatrix} 2 & 4 \\ 5 & 3 \end{vmatrix} - 0\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix}$

$= -3\,(3 - 10) + 0 - 0$

$= -3 \times (-7) = +21$ Ans

(c) $\Delta = \begin{vmatrix} 1 & 4 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 3 \end{vmatrix}$

Expand along $C_3$

$C_3$ के अनुदिश विस्तार

$$\Delta = +3\begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} = 3 \times (5 - 8)$$
$$= 3 \times (-3)$$
$$= -9 \text{ Ans}$$

(d) $\begin{vmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{vmatrix}$

$R_2 \longrightarrow 2R_2$

$$= \frac{1}{2}\begin{vmatrix} 2 & 4 & 6 \\ 2 & 4 & 6 \\ 3 & 1 & 0 \end{vmatrix}$$

$= \frac{1}{2} \times \mathbf{0} = 0 \qquad \{\because R_1 = R_2\}$

(e) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{vmatrix}$

$\because R_1 = R_2 = R_3$ या $C_1 = C_2 = C_3$

$= 0$ Ans

2. $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$

$\Rightarrow 12x - (-14) = 32 - 42$

$\Rightarrow 12x + 14 = -10$

$\Rightarrow 12x = -10 - 14$

$\Rightarrow 12X = -24$

$\Rightarrow X = -2$ Ans

3. $\Delta = \begin{vmatrix} 3 & -2 \\ 4 & 4 \end{vmatrix}$

The Co-factors of the elements of $\Delta$ are-

$\Delta$ के अवयवों के सहखण्ड हैं –

$A_{11} = +(4) = 4 \qquad A_{21} = -(-2) = 2$

$A_{12} = -4 \qquad A_{22} = +(3) = 3$

4. $\Delta = \begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$

$R_1 \longrightarrow R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} 1+w+w^2 & 1+w+w^2 & 1+w+w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 0 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} = 0 \text{ Ans}$$

5. $\begin{vmatrix} x-9 & 1 \\ 7 & 1 \end{vmatrix} = \mathbf{0}$

$\Rightarrow (x-9) \times 1 - 7 \times 1 = 0$

$\Rightarrow x - 9 - 7 = 0$

$\Rightarrow x - 16 = 0$

$\Rightarrow x = 16$ Ans

6. Given equations are :-

दिये गये समीकरण हैं –

$2x - y = 2$

$3x + 4y = 3$

Matrix form (आव्यूह रूप) $Ax = B$

$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ Ans

7. $\begin{vmatrix} 5 & 60 & 7 \\ 3 & 36 & 2 \\ 2 & 24 & 4 \end{vmatrix}$ or $\begin{vmatrix} 5 & 60 & 7 \\ 3 & 36 & 2 \\ 2 & 24 & 4 \end{vmatrix}$

$C_1 \longrightarrow 12C_1$ $\qquad$ $C_2 \longrightarrow C_2 - 12C_1$

$= \begin{vmatrix} 60 & 60 & 7 \\ 36 & 36 & 2 \\ 24 & 24 & 4 \end{vmatrix}$ $\qquad$ $= \begin{vmatrix} 5 & 0 & 7 \\ 3 & 0 & 2 \\ 2 & 0 & 4 \end{vmatrix}$

$\because C_1 = C_2$ $\qquad$ $= 0$

$= 0$

## Short Questions (3 Marks)
## लघुउत्तरीय प्रश्न

**Q.1. Prove that (सिद्ध करें)** $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ **$= (a - b)(b - c)(c - a)$**

**Q.2. Prove that (सिद्ध करें)** $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = \mathbf{0}$

**Q.3. Prove that** $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix}$ **$= (a - b)(b - c)(c - a)$**

**Q.4. Solve for x when, (x के लिए हल करें जब)**

$$\begin{vmatrix} 3+x & 5 & 2 \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix} = 0$$

**Q.5. Find the area of triangle whose vertices are (1, 2), (2, 3), (3, 2)**

**शीर्षों (1, 2), (2, 3), (3, 2) वाले त्रिभुज का क्षेत्रफल ज्ञात करें।**

**Q.6. If** $A = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ -4 & 5 & 3 \end{bmatrix}$**, find adj A**

**Q.7. If** $A = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}$**, verify that A. (adj A) = (adj A) A** $= |A| . I$

**Q.8. Find the inverse of the Matrix** $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$
**आव्यूह A का व्युत्क्रम ज्ञात करें।**

**Q.9. Prove that the Matrix** $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ **satisfies the equation** $x^2 + 4x - 42 = 0$ **and hence find** $A^{-1}$**.**

**सिद्ध करें कि आव्यूह** $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ **समीकरण** $x^2 + 4x - 42 = 0$ **संतुष्ट करता है और** $A^{-1}$ **ज्ञात करें।**

**Q.10. If** $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$**, Show that adj A = 3A'**

**यदि** $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$**, तो सिद्ध करें adj A = 3A'**

## Solutions of short questions (3 Marks)

1. $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$, $R_1 \longrightarrow R_1 - R_2$, $R_2 \longrightarrow R_2 - R_3$

$$= \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$R_1 \longrightarrow \dfrac{R_1}{(a-b)}$, $R_2 \longrightarrow \dfrac{R_2}{(b-c)}$

$$= (a-b)(b-c) \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Expand along $C_1$

($C_1$ के अनुदिश विस्तार)

$$= (a-b)(b-c) \left\{ +1 \begin{vmatrix} 1 & a+b \\ 1 & b+c \end{vmatrix} \right\}$$

$$= (a-b)(b-c) \{ \cancel{b} + c - a - \cancel{b} \}$$

$= (a-b)(b-c)(c-a)$ Proved

2. $\text{L.H.S.} = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

$R_1 \longrightarrow R_1 + R_2 + R_3$

$$= \begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$= 0$

= R.H.S. Proved

3. $\text{L.H.S.} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

$C_1 \longrightarrow C_1 - C_2$, $C_2 \longrightarrow C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ bc-ca & ca-ab & ab \end{vmatrix}$$

$C_1 \longrightarrow \dfrac{C_1}{(a-b)}$, $C_2 \longrightarrow \dfrac{C_2}{(b-c)}$

$$= (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ -c & -a & ab \end{vmatrix}$$

$$= (a-b)(b-c) \left\{ +1 \begin{vmatrix} 1 & 1 \\ -c & -a \end{vmatrix} \right\}$$

$= (a-b)(b-c)(c-a)$

=R.H.S. Proved

4. $\begin{vmatrix} 3+x & 5 & 2 \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix} = 0$

$R_1 \longrightarrow R_1 - R_3$

$$\Rightarrow \begin{vmatrix} 1+x & 0 & -1-x \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix}$$

$C_3 \longrightarrow C_3 + C_1$

$$\Rightarrow \begin{vmatrix} 1+x & 0 & 0 \\ 1 & 7+x & 7 \\ 2 & 5 & 5+x \end{vmatrix} = 0$$

$$\Rightarrow (1+x)\begin{vmatrix} 7+x & 7 \\ 5 & 5+x \end{vmatrix} = 0$$

$$\Rightarrow (1 + x)\{(7 + x))(5 + x) - 35\} = 0$$

$$\Rightarrow (1 + x)(35 + 12x + x^2 - 35) = 0$$

$$\Rightarrow (1 + x)(x^2 + 12x) = 0$$

$$\Rightarrow x(x + 1)(x + 12) = 0$$

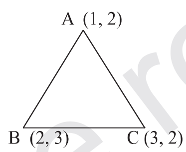
$x = 0, -1, -12$ Ans

5. Given Vertices are (दिए गए शीर्ष हैं)

$A = (1, 2) \qquad x_1 = 1 \quad y_1 = 2$

$B = (2, 3) \qquad x_2 = 2 \quad y_2 = 3$

$C = (3, 2) \qquad x_3 = 3 \quad y_3 = 2$




$$ar(\triangle ABC) = \frac{1}{2}\left\|\begin{matrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{matrix}\right\|$$

$$= \frac{1}{2}\left\|\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 2 \end{matrix}\right\|$$

$$= \frac{1}{2}\left|1(4 - 9) - 1\,(2 - 6) + 1\,(3 - 4)\right|$$

$$= \frac{1}{2}\left|-5 + 4 - 1\right|$$

$$= \frac{1}{2}\left|-2\right|$$

$$= \frac{1}{2} \times 2 = 1 \text{ units}^2 \text{ Ans}$$

6. $A = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ -4 & 5 & 3 \end{bmatrix}$

Determinant (सारणिक) Of A

$$|A| = \begin{vmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ -4 & 5 & 3 \end{vmatrix}$$

The Co-factors of the elements of $|A|$ are

$|A|$ के अवयवों का सहखण्ड हैं :-

$$A_{11} = \begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix} = 6 - 5 = 1, A_{12} = -\begin{vmatrix} 0 & 1 \\ -4 & 3 \end{vmatrix} = -(0 + 4) = -4$$

$$A_{13} = \begin{vmatrix} 0 & 2 \\ -4 & 5 \end{vmatrix} = 0 + 8 = 8$$

$$A_{21} = -\begin{vmatrix} -2 & 4 \\ 5 & 3 \end{vmatrix} = -(-6 - 20) = -(-26) = 26$$

$$A_{22} = +\begin{vmatrix} 1 & 4 \\ -4 & 3 \end{vmatrix} = (3 + 16) = 19$$

$$A_{23} = -\begin{vmatrix} 1 & -2 \\ -4 & 5 \end{vmatrix} = -(5 - 8) = 3$$

$$A_{31} = \begin{vmatrix} -2 & 4 \\ 2 & 1 \end{vmatrix} = (-2 - 8) = -10$$

$$A_{32} = -\begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} = -(1 - 0) = -1$$

$$A_{33} = \begin{vmatrix} 1 & -2 \\ 0 & 2 \end{vmatrix} = (2 - 0) = 2$$

$$\therefore \text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}' = \begin{bmatrix} 1 & -4 & 8 \\ 26 & 19 & 3 \\ -10 & -1 & 2 \end{bmatrix}'$$

$$= \begin{bmatrix} 1 & 26 & -10 \\ -4 & 19 & -1 \\ 8 & 3 & 2 \end{bmatrix} \text{Ans}$$

7. $A = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}, |A| = \begin{vmatrix} 2 & 3 \\ 4 & 8 \end{vmatrix} = 16 - 12 = 4$

$$\text{adj } A = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

Now (अब)

$$A.(\text{adj } A) = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix} . \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 16-12 & -6+6 \\ 32-32 & -12+16 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 4.\ I$$

$$= |A|\,.\,I$$

$$(\text{adj } A).A = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix} . \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 16-12 & 24-24 \\ -8+8 & -12+16 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 4.\ I$$

$$= |A|\,.\,I$$

Clearly $A.(\text{adj } A) = (\text{adj } A)\,.\,A = |A|\,.\,I$ Proved

8. $$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} = -(-1)\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = 9 + 2 = 11$$

The Co-factors of the elements of $|A|$ are

$A_{11} = 0$, $A_{12} = -11$, $A_{13} = 0$

$A_{21} = 3$, $A_{22} = 1$, $A_{23} = -1$

$A_{31} = 2$, , $A_{32} = 8$, $A_{33} = 3$

$$\text{adj } A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

Now, Inverse of A

$$A^{-1} = \frac{1}{|A|}\,.\,\text{adj } A$$

$$= \frac{1}{11}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \text{ Ans.}$$

9. $$A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$$

Given equation (दिया गया समीकरण)

$x^2 + 4x - 4 = 0$

Put $x = A$

$A.A + 4A - 42.I = 0$

$$\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} . \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + 4\,.\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} - 42\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} + \begin{bmatrix} -42 & 0 \\ 0 & -42 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -74 & 20 \\ 8 & -26 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$\Rightarrow 0 = 0$ (True) (सत्य) Proved

Now From equation (1)

$A^2 + 4A - 42.I = 0$

$A^{-1}\,.\,(A^2 + 4A - 42.I) = A^{-1}\,.\,0$

$\Rightarrow A + 4.I - 42A^{-1} = 0$

$\Rightarrow 42A^{-1} = A + 4.I$

$$\Rightarrow 42A^{-1} = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\Rightarrow 42A^{-1} = \begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix} \quad \therefore A^{-1} = \frac{1}{42}\begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix} \text{ Ans}$$

10. $$A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

The Co-factors of the elements of $|A|$ are

$A_{11} = -3$ , $A_{12} = -6$, $A_{13} = -6$

$A_{21} = 6$, $A_{22} = 3$, $A_{23} = -6$

$A_{31} = 6$, , $A_{32} = -6$, $A_{33} = 3$

$$\text{adj } A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$\text{and } A' = \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

$$3\,A' = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

Clearly $\text{adj}A = 3A'$ Proved

## Long Questions (5 Marks)
## दीर्घउत्तरीय प्रश्न

1. **If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, Find $A^{-1}$ and hence solve the system of linear equations.**

**$x + 2y - 3z = -4,\ 2x + 3y + 2z = 2,\ 3x - 3y - 4z = 11$**

यदि $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ तो $A^{-1}$ ज्ञात करें और निम्नलिखित रैखिक समीकरण निकाय का हल करें।

**$x + 2y - 3z = -4,\ 2x + 3y + 2z = 2,\ 3x - 3y - 4z = 11$**

2. **By using elemntry row transformation, find the inverse of the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$**

प्रारंभिक पंक्ति संक्रिया से वर्ग आव्यूह $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ का व्युत्क्रम ज्ञात करें।

3. **Prove that $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ is neither a symmetric matrix nor a skew-symetric matrix.**

**Also express A as the sum of a symmetric Matrix and a skew-symmetric Matrix.**

सिद्ध करें कि $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ न तो सममित है और न ही विषम सममित है। फिर A को एक सममित आव्यूह और विषम सममित आव्यूह के योगफल के रूप में व्यक्त करें।

4. **The sum of three numbers is 6. Twice the third number when added to the first number gives 7 .On adding the sum of the second and third numbers to the thrice the first number we get 12. Find the numbers, using Matrix Method.**

तीन संख्याओं का योग 6 है। जब पहले संख्या में तीसरे संख्या का दो गुना जोड़ने पर 7 प्राप्त होता है। दूसरे तथा तीसरे संख्याओं में पहले संख्या का तीन गुणा जोड़ने पर 12 प्रप्त होता है। तीनों संख्याओं को आव्यूह विधि के द्वारा ज्ञात करें।

5. **Prove that (सिद्ध करें)**

$$\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = \mathbf{4a^2 b^2 c^2}$$

## Solutions of long questions

1. Given system of equations are -

$x + 2y - 3z = -4$ ----------(i)

$2x + 3y + 2z = 2$ ----------(ii)

$3x - 3y - 4z = 11$ ----------(iii)

the given system of equations in matrix form is $AX = B$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix},\ x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix},$$

$= 1(-12 + 6) - 2(-8 - 6) - 3(-6 - 9)$

$= -6 + 28 + 45$

$= -6 + 73$

$= 67 \neq 0$

$\therefore$ A is invertible (A व्युतक्रमणीय है)

So, the system has a unique solution

$x = A^{-1} B$................(1)

Now, the Co-factors of the elements of $|A|$ are

$|A|$ के अवयवों का सहखण्ड हैं :-

$A_{11} = -6,$ $A_{12} = 14,$ $A_{13} = -15$

$A_{21} = 17,$ $A_{22} = 5,$ $A_{23} = 9$

$A_{31} = 13,$ $A_{32} = -8,$ $A_{33} = -1$

$$\therefore \text{ adj } A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

and $A^{-1} = \dfrac{1}{|A|}$ adj A

$$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

Now from equation (1) (अब समीकरण (1) से)

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}.\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}.\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}.\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

So, $x = 3, y = -2, z = 1$ Ans

2. Given, $\mathbf{A} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

we know A= I.A

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

$R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

$R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

$R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

$R_2 \longleftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & -2 & -5 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 0 \end{bmatrix}.A$$

$R_1 \rightarrow R_1 - R_2, R_3 \rightarrow R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix}.A$$

$R_1 \rightarrow R_1 + R_3, R_2 \rightarrow R_2 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.A$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ Ans

3. Given, $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

Now (अब) $A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Here (यहाँ) $A \neq A' \Rightarrow$ Not symmetric Matrix
सममित आव्यूह नहीं है।

$A \neq -A' \Rightarrow$ Not Skew-symmetric Matrix
विषम सममित आव्यूह नहीं है।

Symmetric Matrix $= \frac{1}{2}(A + A')$
(सममित आव्यूह)

$$= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

Skew-symmetric Matrix $= \frac{1}{2}(A - A')$
(विषम सममित आव्यूह)

$$= \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

square Matrix, A = symmetric Matrix + Skew symmetric Matrix

$$A = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

Ans

4. Let First number = x
पहली संख्या = x

Second Number = y

दूसरी संख्या = y

Third Number = z

तीसरी संख्या = z

By question (प्रश्न से)

$x + y + z = 6$ ---------(1)

$x + 2z = 7$ ----------(2)

$3x + y + z = 12$ -------(3)

The Given system in Matrix form is AX =B

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix}$$

$|A| = 1(0 - 2) - 1(1 - 6) + 1(1 - 0)$

$= -2 + 5 + 1$

$|A| = 4 \neq 0$ $\therefore$ A is invertible (A व्युत्क्रमणीय है।)

Now $X = A^{-1}B$ ---------(4)

The Co-factors of the elements of $|A|$ are-

$A_{11} = -2$, $A_{12} = 5$, $A_{13} = 1$

$A_{21} = 0$, $A_{22} = -2$, $A_{23} = 2$

$A_{31} = 2$, $A_{32} = -1$, $A_{33} = -1$

$$\therefore \text{ adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$\text{Now } A^{-1} = \frac{1}{|A|} \text{ adj } A$$

$$= \frac{1}{4} \cdot \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$\therefore$ By (4), $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \cdot \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4} \cdot \begin{bmatrix} -12+0+24 \\ 30-14-12 \\ 6+14-12 \end{bmatrix}$$

$$= \frac{1}{4} \cdot \begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ $x = 3, y = 1, z = 2$

$\therefore$ Required numbers = 3, 1, 2 Ans

(अभिष्ट संख्याएँ)

5. $$\text{L.H.S.} = \begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

$C_1 \rightarrow \frac{C_1}{a}$, $C_2 \rightarrow \frac{C_2}{b}$, $C_3 \rightarrow \frac{C_3}{c}$,

$$= abc \begin{vmatrix} a & c & c+a \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$R_2 \rightarrow R_2 - (R_1 + R_3)$

$$= abc \begin{vmatrix} a & c & c+a \\ 0 & -2c & -2c \\ b & b+c & c \end{vmatrix}$$

$C_2 \rightarrow C_2 - C_3$

$$= abc \begin{vmatrix} a & -a & c+a \\ 0 & 0 & -2c \\ b & b & c \end{vmatrix}$$

Expand along $R_2$

($R_2$ के अनुदिश विस्तार करने पर)

$$= abc \cdot 2c \begin{vmatrix} a & -a \\ b & b \end{vmatrix}$$

$= 2abc^2 \cdot (ab + ab)$

$= 2abc^2 \times 2ab$

$= 4\, a^2b^2c^2$

= R.H.S. Proved