Chapter : 7	INTEGRALS समाकलन

MCQ :

1. $\int \sin 2x\, dx$ **is equal to** (बराबर हैं)

a) $2\cos 2x + c$ b) $\frac{1}{2}\cos 2x + c$

c) $-\frac{1}{2}\cos 2x + c$ d) $\cos 2x + c$

2. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$ **is equal to** (बराबर हैं)

a) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + c$ b) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^{2} + c$

c) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$ d) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + c$

3. $\int \cot x\, dx =$

a) $\log|\sin x| + c$ b) $\log|\cos x| + c$

c) $\log|\sec x| + c$ d) None of these (इनमें से कोई नहीं)

4. $\int \frac{dx}{\sin^2 x \cos^2 x} =$

a) $\tan x + \cot x + c$

b) $\tan x - \cot x + c$

c) $\tan x . \cot x + c$

d) $\tan x - \cot 2x + c$

5. $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ **equals** (बराबर है)

a) $10^x - x^{10} + c$ b) $10^x + x^{10} + c$

c) $(10^x - x^{10})^{-1} + c$ d) $\log(10^x + x^{10}) + c$

6. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ **is equal to** (बराबर है)

a) $\tan x + \cot x + c$

b) $\tan x + \operatorname{cosec} x + c$

c) $-\tan x + \cot x + c$

d) $\tan x + \sec x + c$

7. $\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx =$

a) $-\cot(ex^x) + c$ b) $\tan(xe^x) + c$

c) $\tan(e^x) + c$ d) $\cot(e^x) + c$

8. $\int \frac{dx}{x^2 + 2x + 2}$ **equals** (बराबर है)

a) $x\tan^{-1}(x+1) + c$

b) $\tan^{-1}(x+1) + c$

c) $(x+1)\tan^{-1}x + c$

d) $\tan^{-1}x + c$

9. $\int \frac{dx}{\sqrt{9x - 4x^2}}$ **equals** (बराबर है)

a) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right) + c$

b) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right) + c$

c) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right) + c$

d) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right) + c$

10. $\int \frac{x\,dx}{(x-1)(x-2)}$ **equals** (बराबर हैं)

a) $\log\left|\frac{(x-1)^2}{x-2}\right| + c$

b) $\log\left|\frac{(x-2)^2}{x-1}\right| + c$

c) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + c$

d) $\log|(x-1)(x-2)| + c$

11. $\int \frac{dx}{x(x^2+1)}$ **equals** (बराबर है)

a) $\log|x| - \frac{1}{2}\log(x^2+1) + c$

b) $\log|x| + \frac{1}{2}\log(x^2+1) + c$

c) $-\log|x| + \frac{1}{2}\log(x^2+1) + c$

d) $\frac{1}{2}\log|x| + \log(x^2+1) + c$

12. $\int x^2 e^{x^3} dx$ **equals** (बराबर है)

a) $\frac{1}{3}e^{x^3} + c$

b) $\frac{1}{3}e^{x^2} + c$

c) $\frac{1}{2}e^{x^3} + c$

d) $\frac{1}{2}e^{x^2} + c$

13. $\int e^x \sec x(1 + \tan x)\, dx =$

a) $e^x\cos x+c$ b) $e^x\sec x+c$
c) $e^x\sin x+c$ d) $e^x\tan x+c$

14. $\int\sqrt{1+x^2}\,dx$ **is equal to** (बराबर है)

a) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+c$
b) $\frac{2}{3}(1+x^2)^{\frac{3}{2}}+c$
c) $\frac{2}{3}x(1+x^2)^{\frac{3}{2}}+c$
d) $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+c$

15. $\int\frac{1}{x+\sqrt{x}}dx$ **is equal to** (बराबर है)

a) $\log|x|+\log(1+\sqrt{x})+c$
b) $2\log(1+\sqrt{x})+c$
c) $\log(1+\sqrt{x})+c$
d) $\log|x|+c$

16. $\int\frac{x^4+1}{x^2+1}dx=$

a) $\frac{x^3}{3}+x+\tan^{-1}x+c$
b) $\frac{x^3}{3}-x+\tan^{-1}x+c$
c) $\frac{x^3}{3}+x+2\tan^{-1}x+c$
d) $\frac{x^3}{3}-x+2\tan^{-1}x+c$

17. $\int\frac{dx}{e^x+e^{-x}}$ **is equal to** (बराबर है)

a) $\tan^{-1}(e^{x})+c$
b) $\tan^{-1}(e^{-x})+c$
c) $\log(e^x-e^{-x})+c$
d) $\log(e^x+e^{-x})+c$

18. $\int\frac{\cos 2x}{(\sin x+\cos x)^2}dx=$

a) $\frac{-1}{\sin x+\cos x}+c$
b) $\log|\sin x+\cos x|+c$
c) $\log|\sin x-\cos x|+c$
d) $\frac{1}{(\sin x+\cos x)^2}+c$

19 $\int\frac{e^x}{e^{2x}+1}dx$ **is equal to** (बराबर है)

a) $\log(e^x+e^{-x})+c$ b) $\tan^{-1}e^x+c$
c) $\log(e^{2x}+1)+c$ d) $\tan^{-1}(2e^x)+c$

20. $\int\cos\sqrt{x}\,dx$ **equals** (बराबर है)

a) $\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}+c$
b) $\sqrt{x}\sin\sqrt{x}-\cos\sqrt{x}+c$
c) $\frac{1}{2}\left[\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}\right]+c$
d) $2\left[\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}\right]+c$

21. $\int e^x(1+\tan x+\tan^2 x)dx=$

a) $e^x\cos x+c$ b) $e^x\sin x+c$
c) $e^x\tan x+c$ d) $e^x\sec x+c$

22. $\int\frac{dx}{1+x^2}=$

a) $\sin^{-1}x+c$ b) $\tan^{-1}x+c$
c) $\log(1+x^2)+c$ d) $\cot^{-1}x+c$

23. $\int\frac{\cos 2x+2\sin^2 x}{\cos^2 x}dx=$

a) $\tan x+c$ b) $\cot x+c$
c) $\sin x+c$ d) $\cos x+c$

24. $\int\frac{dx}{\sqrt{1-x^2}}$ **equals** (बराबर है)

a) $\cos^{-1}x+c$
b) $-\sin^{-1}x+c$
c) $\sin^{-1}x+c$
d) $\tan^{-1}x+c$

25. $\int e^x(\sin x+\text{cox})dx=$

a) $e^x\sin x+c$ b) $e^x\cos x+c$
c) $e^x\tan x+c$ d) $\frac{e^x}{\sin x}+C$

26. $\int\frac{\sec^2 x}{\operatorname{cosec}^2 x}dx=$

a) $\cot x+x+c$ b) $\cot x-x+c$
c) $\tan x-x+c$ d) $\tan x+c$

27. $\int\frac{2}{\sqrt{1-4x^2}}dx=$

a) $\tan^{-1}(2x)+c$ b) $\cot^{-1}(2x)+c$
c) $\cos^{-1}(2x)+c$ d) $\sin^{-1}(2x)+c$

28. $\int_{-\pi/2}^{\pi/2}\sin^9 x\,dx=$

a) -1 b) 0
c) 1 c) None of these/ (इनमें से कोई नहीं)

29. $\int_0^p \frac{\sqrt{x}}{\sqrt{x}+\sqrt{p-x}}\,dx =$

a) p b) p/2
c) 2p d) none of these/ (इनमें से कोई नहीं)

30. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals (बराबर है)

a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$
c) $\frac{\pi}{6}$ d) $\frac{\pi}{12}$

31. $\int_0^{2/3} \frac{dx}{4+9x^2}$ equals (बराबर है)

a) $\frac{\pi}{6}$ b) $\frac{\pi}{12}$
c) $\frac{\pi}{24}$ d) $\frac{\pi}{4}$

32. $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}\,dx$, is equal to (बराबर है)

a) 6 b) 0
c) 3 d) 4

33. $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\,dx =$

a) 0 b) 2
c) π d) 1

34. $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx =$

a) 2 b) $\frac{3}{4}$
c) 0 d) -2

35. $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx =$

a) 1 b) 0
c) -1 d) $\frac{\pi}{4}$

36. $\int_0^1 \sqrt{x(1-x)}\,dx =$

a) $\frac{\pi}{8}$ b) $\frac{\pi}{6}$
c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$

37. $\int_1^{\sqrt{3}} \frac{1}{1+x^2}\,dx =$

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$
c) $\frac{\pi}{6}$ d) $\frac{\pi}{12}$

38. $\int_0^1 e^x\,dx =$

a) e - 1 b) $\frac{e-1}{e}$
c) $\frac{e^2-1}{e}$ d) $\frac{e^2-1}{2}$

39. $\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}} =$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$
c) $\frac{\pi}{2}$ d) None of these/ (इनमें से कोई नहीं)

40. $\int_0^1 \frac{x}{1+x}\,dx =$

a) 1-log2 b) 2
c) 1+log2 d) log2

ANSWER

1.	c	21.	c
2.	c	22.	b
3.	a	23.	a
4.	b	24.	c
5.	d	25.	a
6.	a	26.	c
7.	b	27.	d
8.	b	28.	b
9.	b	29.	b
10.	b	30.	d
11.	a	31.	c
12.	a	32.	d
13.	b	33.	c
14.	a	34.	c
15.	b	35.	b
16.	d	36.	a
17.	a	37.	d
18.	b	38.	a
19.	b	39.	a
20.	d	40.	a

INTEGRALS (02 MARKS)

EVALUATE (मान निकाले)

1) $\int \tan x \, dx$

2) $\int \sec x \, dx$

3) $\int \operatorname{cosec} x \, dx$

4) $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

5) $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

6) $\int (1-x)\sqrt{x} \, dx$

7) $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

8) $\int \frac{\sin(\tan^{-1}x)}{1+x^2} dx$

9) $\int \sin^3 x \cos^2 x \, dx$

10) $\int \frac{1}{1+\tan x} dx$

11) $\int \frac{\sin x}{\sin(x+a)} dx$

12) $\int \frac{(\log x)^2}{x} dx$

13. $\int \frac{1}{x + x\log x} dx$

14. $\int (x^3-1)^{1/3} x^5 \, dx$

15. $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$

16. $\int \frac{e^{2x}-1}{e^{2x}+1} dx$

17. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx$

18. $\int \sec^2(7-4x) \, dx$

19. $\int \frac{1}{\cos^2 x(1-\tan x)^2} dx$

20. $\int \frac{(x+1)(x+\log x)^2}{x} dx$

21. $\int \frac{1-\cos x}{1+\cos x} dx$

22. $\int \frac{\sin^2 x}{1+\cos x} dx$

23. $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

24. $\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$

25. $\int \frac{3x^2}{x^6+1} dx$

26. $\int \frac{x^2}{1-x^6} dx$

27. $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$

28. $\int \frac{1}{x^2-9} dx$

29. $\int \frac{1}{e^x-1} dx$

30. $\int \log x \, dx$

31. $\int x e^x \, dx$

32. $\int x \sin x \, dx$

33. $\int \tan^{-1} x \, dx$

34. $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

35. $\int \sqrt{4-x^2}\,dx$

36. $\int \sqrt{x^2+4x+1}\,dx$

37. $\int \frac{1}{x-x^2}\,dx$

38. $\int \sqrt{\frac{x}{1-x}}\,dx$

39. $\int \frac{(\sin^{-1}x)^3}{\sqrt{1-x^2}}\,dx$

40. $\int \frac{dx}{1+e^x}$

DEFINITE INTEGRALS (2 Marks)

1. $\int_{-1}^{1} (x+1)\,dx$

2. $\int_{2}^{3} \frac{1}{x}\,dx$

3. $\int_{0}^{\pi/2} \cos 2x\,dx$

4. $\int_{4}^{5} e^x\,dx$

5. $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$

6. $\int_{2}^{3} \frac{dx}{x^2-1}$

7. $\int_{0}^{\pi/2} \cos^2 x\,dx$

8. $\int_{2}^{3} \frac{x}{x^2+1}\,dx$

9. $\int_{0}^{1} \frac{\tan^{-1}x}{1+x^2}\,dx$

10. $\int_{-\pi/4}^{\pi/4} \sin^2 x\,dx$

11. $\int_{0}^{1} x(1-x)^n\,dx$

12. $\int_{-1}^{1} e^x\,dx$

INDEFINITE INTEGRATION (3 MARKS)

EVALUATE (मान निकाले)

1. $\int (5x+3)\sqrt{2x-1}\,dx$

2. $\int \frac{x(x+3)}{x+5}\,dx$

3. $\int \sin^4 x\,dx$

4. $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}\,dx$

5. $\int \tan^4 x\,dx$

6. $\int \tan^3 x\,dx$

7. $\int \frac{dx}{x^2-a^2}$

8. $\int \frac{dx}{a^2-x^2}$

9. $\int \frac{dx}{x^2+a^2}$

10. $\int \frac{dx}{\sqrt{x^2-a^2}}$

11. $\int \frac{dx}{\sqrt{a^2-x^2}}$

12. $\int \frac{dx}{\sqrt{x^2+a^2}}$

13. $\int \frac{dx}{x^2-6x+13}$

14. $\int \frac{x+3}{\sqrt{5-4x-x^2}}dx$

15. $\int \frac{x+2}{\sqrt{x^2-1}}dx$

16. $\int \frac{x}{(x-1)(x-2)(x-3)}dx$

17. $\int e^x \sin x\, dx$

18. $\int x \sin^{-1}x\, dx$

19. $\int \frac{(x^2+1)e^x}{(x+1)^2}dx$

20. $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

21. $\int \sqrt{x^2+2x+5}\,dx$

22. $\int \sqrt{1+3x-x^2}\,dx$

23. $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$

DEFINITE INTEGRALS (03 MARKS)

1. $\int_0^1 \frac{dx}{(\sqrt{x+1}+\sqrt{x})}$

2. $\int_0^{\pi/2} \frac{dx}{1+\sin x}$

3. $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

4. $\int_0^1 |x-1|dx$

5. $\int_0^{\pi/2} \log \sin x\, dx$

6. $\int_0^{\pi/4} \log(1+\tan x)\,dx$

7. $\int_0^{2\pi} |\cos x|dx$

8. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

9. $\int_0^{2\pi} \cos^5 x\, dx$

10. $\int_0^{\pi} \frac{x}{1+\sin x}dx$

11. $\int_0^{\pi/2} \frac{x}{\sin x+\cos x}dx$

12. $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}dx$

DEFINITE INTEGRALS AS THE LIMIT OF A SUM (05 MARKS)

EVALUATE THE FOLLOWING DEFINITE INTEGRALS AS THE LIMIT OF SUM

1. $\int_0^1 x^2 dx$

2. $\int_0^1 e^x dx$

3. $\int_0^3 x\, dx$

4. $\int_0^4 (x+e^{2x})dx$

SOLUTION OF INTEGRATION(2 MARKS)

1. $Let\ \ I = \int \tan x\, dx = \int \frac{\sin x}{\cos x}dx$

put $\cos x = t$ so that $-\sin x dx = dt$

$\therefore \sin x\, dx = -dt$

$$\int \tan x dx = -\int \frac{dt}{t}$$
$$= -\log|t| + C$$
$$= -\log|\cos x| + c$$
$$= \log|\sec x| + c$$

$$\therefore \int \tan x\, dx = -\log|\cos x| + c$$
$$= \log|\sec x| + c$$

2. *Let* $I = \int \sec x \, dx$

$= \int \frac{\sec x(\sec x+\tan x)}{\sec x+\tan x} dx$

put $\sec x + \tan x = t$

$\sec x (\sec x + \tan x) dx = dt$

$\therefore I = \int \frac{dt}{t} = \log|t| + c$

$= \log|\sec x + \tan x| + c$

3. Let $I = \int \operatorname{cosec} x \, dx$

$= \int \frac{\operatorname{cosec} x(\operatorname{cosec} x-\cot x)}{(\operatorname{cosec} x-\cot x)} dx$

Put $\operatorname{cosec} x - \cot x = t$

$\therefore \operatorname{cosec} x(\operatorname{cosec} x - \cot x) dx = dt$

$\Rightarrow I = \int \frac{dt}{t} = \log|t| + c$

$= \log|\operatorname{cosec} x - \cot x| + c.$

4. *Let* $I = \int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx = \int \left(x + \frac{1}{x} - 2\right) dx$

$= \int x \, dx + \int \frac{1}{x} dx - \int 2 \, dx$

$= \frac{x^2}{2} + \log|x| - 2x + c$

5. $I = \int \frac{x^3-x^2+x-1}{x-1} dx$

$= \int \frac{x^2(x-1)+1(x-1)}{x-1} dx$

$= \int \frac{(x-1)(x^2+1)}{x-1} dx$

$= \int (x^2+1) dx = \frac{x^3}{3} + x + c$

6. $I = \int (1-x)\sqrt{x} \, dx = \int (\sqrt{x} - x\sqrt{x}) dx$

$= \int x^{1/2} dx - \int x^{3/2} dx$

$= \frac{x^{1/2+1}}{\frac{1}{2}+1} - \frac{x^{3/2+1}}{\frac{3}{2}+1} + c$

$= \frac{2}{3} x^{3/2} - \frac{2}{5} x^{5/2} + c$

7. $I = \int \frac{2-3\sin x}{\cos^2 x} dx = \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right) dx$

$= 2\int \sec^2 x \, dx - 3\int \sec x \, . \tan x \, dx$

$= 2\tan x - 3\sec x + c$

8. $I = \int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$

Put $\tan^{-1} x = t$, $\therefore \frac{1}{1+x^2} dx = dt$

$\therefore I = \int \sin t \, dt = -\cos(t) + c$

$= -\cos(\tan^{-1} x) + c$

9. $I = \int \sin^3 x \cos^2 x \, dx = \int \sin^2 x \cos^2 x \sin x \, dx$

$= \int (1 - \cos^2 x)\cos^2 x(\sin x) \, dx;$

put $\cos x = t$

$\Rightarrow -\sin x dx = dt$

$\therefore I = -\int (1-t^2)t^2 dt = -\int (t^2 - t^4) dt$

$= -\left(\frac{t^3}{3} - \frac{t^5}{5}\right) + c$

$= -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + c.$

10. $I = \int \frac{1}{1+\tan x} dx = \int \frac{\cos x}{\cos x+\sin x} dx$

$= \frac{1}{2}\int \frac{2\cos x}{\cos x+\sin x} dx$

$= \frac{1}{2}\int \frac{\cos x+\sin x+\cos x-\sin x}{\cos x+\sin x} dx$

$= \frac{1}{2}\int 1 dx + \frac{1}{2}\int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

$= \frac{1}{2} x + \frac{1}{2}\log|\cos x + \sin x| + c$

11. *Let* $I = \int \frac{\sin x}{\sin(x+a)} dx$

put $(x+a) = t \Rightarrow dx = dt$

$\therefore I = \int \frac{\sin(t-a)}{\sin t} dt$

$= \int \frac{\sin t \cos a - \cos t \sin a}{\sin t} dt$

$= \cos a \int dt - \int \sin a \cot t \, dt$

$= \cos a(t) - \sin a \int \cot t \, dt$

$= \cos a(t) - \sin a \log|\sin t| + c$

$= \cos a(x+a) - \sin a \log|\sin(x+a)| + c$

$= x\cos a + a\cos a - \sin a \log|\sin(x+a)| + c$

12. let $I = \int \frac{(\log x)^2}{x} dx$

put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$\therefore I = \int t^2 dt = \frac{t^3}{3} + c$

$= \frac{(\log x)^3}{3} + c$

13. $$I=\int \frac{1}{x+x\log x}dx=\int \frac{1}{x(1+\log x)}dx$$
put $1+\log x = t \Rightarrow \frac{1}{x}dx = dt$
$$\therefore I=\int \frac{dt}{t}=\log|t|+c$$
$$=\log|1+\log x|+c.$$

14. $$I=\int (x^3-1)^{1/3}x^5dx=\int (x^3-1)^{\frac{1}{3}}x^3x^2dx$$
put $x^3-1=t \Rightarrow 3x^2dx=dt \Rightarrow x^2dx=\frac{1}{3}dt$
$$\therefore I=\frac{1}{3}\int t^{1/3}(t+1)dt=\frac{1}{3}\int (t^{4/3}+t^{1/3})dt$$
$$=\frac{1}{3}\left[\frac{t^{4/3+1}}{\frac{4}{3}+1}\right]+\frac{1}{3}\left[\frac{t^{1/3+1}}{\frac{1}{3}+1}\right]+c$$
$$=\frac{1}{7}t^{7/3}+\frac{1}{4}t^{4/3}+c$$
$$=\frac{1}{7}(x^3-1)^{7/3}+\frac{1}{4}(x^3-1)^{4/3}+c$$

15. $$I=\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$$
put $\tan^{-1}x=t \Rightarrow \frac{1}{1+x^2}dx=dt$
$$\therefore I=\int e^t dt \quad =e^t+c$$
$$=e^{\tan^{-1}x}+c$$

16. $$I=\int \frac{e^{2x}-1}{e^{2x}+1}dx=\int \frac{e^{2x}-1}{e^x}\Big/\frac{e^{2x}+1}{e^x}dx$$
$$=\int \frac{e^x-e^{-x}}{e^x+e^{-x}}dx$$
put $e^x+e^{-x}=t \Rightarrow (e^x-e^{-x})dx=dt$
$$\therefore I=\int \frac{dt}{t}=\log|t|+c=\log|e^x+e^{-x}|+c$$

17. $$I=\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$$
put $\sin^{-1}x=t \Rightarrow \frac{1}{\sqrt{1-x^2}}dx=dt$
$$\therefore I=\int t\,dt=\frac{t^2}{2}+c=\frac{(\sin^{-1}x)^2}{2}+c$$

18. $$I=\int \sec^2(7-4x)dx$$
put $7-4x=t \Rightarrow -4dx=dt$
$$\therefore dx=-\frac{1}{4}dt$$
$$\therefore I=-\frac{1}{4}\int \sec^2 t\,dt=-\frac{1}{4}\tan t+c$$
$$=-\frac{1}{4}\tan(7-4x)+c$$

19. $$I=\int \frac{1}{\cos^2x(1-\tan x)^2}dx=\int \frac{\sec^2x}{(1-\tan x)^2}dx$$
put $1-\tan x=t \Rightarrow -\sec^2x\,dx=dt$
$$\Rightarrow \sec^2x\,dx=-dt$$
$$I=-\int \frac{dt}{t^2}=\frac{1}{t}+c=\frac{1}{(1-\tan x)}+c$$

20. $$I=\int \frac{(x+1)(x+\log x)^2}{x}dx$$
$$=\int \left(\frac{x+1}{x}\right)(x+\log x)^2dx$$
$$=\int \left(1+\frac{1}{x}\right)(x+\log x)^2dx$$
put $x+\log x=t \Rightarrow \left(1+\frac{1}{x}\right)dx=dt$
$$\therefore I=\int t^2dt=\frac{t^3}{3}+c=\frac{(x+\log x)^3}{3}+c$$

21. $$I=\int \frac{1-\cos x}{1+\cos x}dx=\int \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$$
$$=\int \tan^2\frac{x}{2}dx=\int \left(\sec^2\frac{x}{2}-1\right)dx$$
$$=\frac{\tan\frac{x}{2}}{1/2}-x+c$$
$$=2\tan\frac{x}{2}-x+c$$

22. $$I=\int \frac{\sin^2x}{1+\cos x}dx=\int \frac{1-\cos^2x}{1+\cos x}dx$$
$$=\int \frac{(1+\cos x)(1-\cos x)}{1+\cos x}dx=\int (1-\cos x)dx$$
$$=x-\sin x+c$$

23. $$I=\int \frac{\cos 2x+2\sin^2x}{\cos^2x}dx$$
$$=\int \frac{1-2\sin^2x+2\sin^2x}{\cos^2x}dx=\int \frac{1}{\cos^2x}dx$$
$$=\int \sec^2x\,dx=\tan x+c$$

24. $$I=\int \frac{\cos 2x}{(\cos x+\sin x)^2}dx=\int \frac{\cos^2x-\sin^2x}{(\cos x+\sin x)^2}dx$$
$$=\int \frac{(\cos x+\sin x)(\cos x-\sin x)}{(\cos x+\sin x)^2}dx$$
$$=\int \frac{\cos x-\sin x}{\cos x+\sin x}dx$$
$$=\int \frac{1-\tan x}{1+\tan x}dx$$
$$=\int \tan\left(\frac{\pi}{4}-x\right)dx$$
$$=\log\left|\cos\left(\frac{\pi}{4}-x\right)\right|+c$$

or,

$$I=\int \frac{\cos 2x}{(\cos x+\sin x)^2}dx=\int \frac{\cos^2x-\sin^2x}{(\cos x+\sin x)^2}dx$$
$$=\int \frac{(\cos x+\sin x)(\cos x-\sin x)}{(\cos x+\sin x)^2}dx$$
$$=\int \frac{\cos x-\sin x}{\cos x+\sin x}dx$$
put $\cos x+\sin x=t$
$$\Rightarrow (\cos x-\sin x)dx=dt$$
$$\therefore I=\int \frac{dt}{t}=\log|t|+c$$
$$=\log|\cos x+\sin x|+c$$

25. $I=\int \frac{3x^2}{x^6+1}dx = \int \frac{3x^2}{(x^3)^2+1}dx$

put $x^3=t \quad \Rightarrow 3x^2dx=dt$

$\therefore I=\int \frac{dt}{t^2+1}=\tan^{-1}(t)+c$

$=\tan^{-1}(x^3)+c$

26. $I=\int \frac{x^2}{1-x^6}dx = \int \frac{x^2}{1-(x^3)^2}dx$

put $x^3=t \quad \Rightarrow 3x^2dx=dt$

$x^2dx=\frac{1}{3}dt$

$\therefore I=\frac{1}{3}\int \frac{dt}{1-t^2} \quad =\frac{1}{3}\int \frac{dt}{(1)^2-(t)^2}$

$=\frac{1}{3}\times\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|+c$

$=\frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right|+c$

27. $I=\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}dx$

put $\tan x=t \quad \Rightarrow \sec^2 x\,dx=dt$

$\therefore I=\int \frac{dt}{\sqrt{t^2+4}}=\int \frac{dt}{\sqrt{t^2+2^2}}$

$=\log\left|t+\sqrt{t^2+4}\right|+c$

$=\log\left|\tan x+\sqrt{\tan^2 x+4}\right|+c.$

28. $I=\int \frac{1}{x^2-9}dx \quad =\int \frac{1}{x^2-3^2}dx$

$=\frac{1}{2\times3}\log\left|\frac{x-3}{x+3}\right|+c$

$=\frac{1}{6}\log\left|\frac{x-3}{x+3}\right|+c.$

29. $I=\int \frac{1}{e^x-1}dx$

put $e^x-1=t \quad \Rightarrow e^x dx=dt$

$\therefore dx=\frac{1}{e^x}dt \quad =\frac{1}{(t+1)}dt$

$\therefore I=\int \frac{1}{t(t+1)}dt=\int \frac{t+1-t}{t(t+1)}dt$

$=\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt \quad =\log|t|-\log|t+1|+c$

$=\log\left|\frac{t}{t+1}\right|+c=\log\left|\frac{e^x-1}{e^x}\right|+c$

30. $I=\int \log x\,dx=\int \log x\,.1dx$

$=\log x\int 1dx-\int\left[\frac{d}{dx}(\log x)\int 1dx\right]dx$

$=\log x.x-\int \frac{1}{x}.\,x\,dx$

$=x\log x-\int 1\,dx=x\log x-x+c$

$=x(\log x-1)+c$

31. $I=\int xe^x dx$

$=x\int e^x dx-\int\left[\frac{d}{dx}(x)\int e^x dx\right]dx$

$=xe^x-\int 1.e^x dx$

$=xe^x-e^x+c$

$=(x-1)e^x+c$

32. $I=\int x\sin x\,dx$

$=x\int \sin x dx-\int\left[\frac{d}{dx}(x)\int \sin x dx\right]dx$

$=x(-\cos x)-\int 1.(-\cos x)\,dx$

$=-x\cos x+\int \cos x\,dx$

$=-x\cos x+\sin x+c$

33. $I=\int \tan^{-1}x\,dx=\int(\tan^{-1}x.1)\,dx$

$=\tan^{-1}x.\int 1dx-\int\left[\frac{d}{dx}(\tan^{-1}x)\int 1dx\right]dx$

$=x\tan^{-1}x-\int \frac{x}{1+x^2}dx$

$=x\tan^{-1}x-\frac{1}{2}\int \frac{2x}{1+x^2}dx$

$=x\tan^{-1}x-\frac{1}{2}\log|1+x^2|+c$

34. $I=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$

consider $f(x)=\frac{1}{x}$, then $f'(x)=-\frac{1}{x^2}$

thus the given integrand is of

the form $e^x[f(x)+f'(x)]$.

$\therefore I=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx=e^x.\frac{1}{x}+c$

$\therefore I=\frac{e^x}{x}+c.$

35. $I=\int \sqrt{4-x^2}\,dx$

$=\int \sqrt{2^2-x^2}\,dx$

$=\frac{x}{2}\sqrt{2^2-x^2}+\frac{2^2}{2}\sin^{-1}\frac{x}{2}+c$

$=\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+c$

36. $I=\int \sqrt{x^2+4x+1}\,dx$

$=\int \sqrt{x^2+4x+4-4+1}\,dx$

$=\int \sqrt{(x+2)^2-3}\,dx$

$=\int \sqrt{(x+2)^2-(\sqrt{3})^2}\,dx$

$=\frac{x+2}{2}\sqrt{(x+2)^2-(\sqrt{3})^2}-$

$\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+c$

$=\frac{x+2}{2}\sqrt{x^2+4x+1}-$

$\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+c$

37. $I=\int \frac{1}{x-x^2}dx=\int \frac{1}{x(1-x)}dx$

$=\int (\frac{1}{x}+\frac{1}{1-x})\,dx$

$=\log|x|-\log|1-x|+c$

$=\log\left|\frac{x}{1-x}\right|+c$

38. $I=\int \sqrt{\frac{x}{1-x}}\,dx$

put $x=\sin^2 t$

$dx=2\sin t\cos t\,dt$

$=\int \sqrt{\frac{\sin^2 t}{1-\sin^2 t}}\cdot 2\sin t\cos t\,dt$

$=\int \frac{\sin t}{\cos t}\cdot 2\sin t\cos t\,dt$

$=\int 2\sin^2 t\,dt$

$=\int (1-\cos 2t)\,dt=t-\frac{\sin 2t}{2}+c$

$=t-\frac{2\sin t.\cos t}{2}+c$

$=t-\sin t.\cos t+c$

$=\sin^{-1}\sqrt{x}-\sqrt{x}.\sqrt{1-x}+c$

39. $I=\int \frac{(\sin^{-1}x)^3}{\sqrt{1-x^2}}dx$

put $\sin^{-1}x=t$

$\therefore \frac{1}{\sqrt{1-x^2}}dx=dt$

$\therefore I=\int t^3\,dt=\frac{t^4}{4}+c$

$=\frac{(\sin^{-1}x)^4}{4}+c$

40. $I=\int \frac{1}{1+e^x}dx$

put $1+e^x=t \Rightarrow e^x dx=dt$

$\Rightarrow dx=\frac{1}{e^x}dt$

$\Rightarrow dx=\frac{1}{t-1}dt$

$\therefore I=\int \frac{1}{t(t-1)}dt=\int (\frac{1}{t-1}-\frac{1}{t})\,dt$

$=\log|t-1|-\log|t|+c$

$=\log\left|\frac{t-1}{t}\right|+c$

$=\log\left|\frac{e^x}{1+e^x}\right|+c$

DEFINITE INTEGRATION (2 MARKS)

1. $I=\int_{-1}^{1}(x+1)\,dx=\left[\frac{x^2}{2}+x\right]_{-1}^{1}$

$=\frac{1}{2}\left[(1)^2-(-1)^2\right]+\left[1-(-1)\right]$

$=\frac{1}{2}(1-1)+(1+1)$

$=\frac{1}{2}\times 0+2=2$

2. $I=\int_2^3 \frac{1}{x}dx=[\log x]_2^3$

$=(\log 3-\log 2)$

$=\log\frac{3}{2}$

3. $I=\int_0^{\pi/2}\cos x dx=[\sin x]_0^{\pi/2}$

$=\sin\frac{\pi}{2}-\sin 0=1-0=1$

4. $I=\int_4^5 e^x dx=[e^x]_4^5$

$=e^5-e^4$

$=e^4(e-1).$

5. $I=\int_0^1 \frac{dx}{\sqrt{1-x^2}}=[\sin^{-1}x]_0^1$

$=\sin^{-1}1-\sin^{-1}0$

$=\frac{\pi}{2}-0=\frac{\pi}{2}$

6. $I=\int_2^3 \frac{dx}{x^2-1}=\frac{1}{2}\left[\log\frac{x-1}{x+1}\right]_2^3$

$=\frac{1}{2}\left[\log(x-1)-\log(x+1)\right]_2^3$

$=\frac{1}{2}\left[\log 2-\log\frac{4}{3}\right]$

$=\frac{1}{2}\log\frac{2}{\frac{4}{3}}=\frac{1}{2}\log\frac{3}{2}$

7. $I=\int_0^{\pi/2}\cos^2 x dx$ -------- (i)

$=\int_0^{\pi/2}\cos^2(\tfrac{\pi}{2}-x)\,dx$

$=\int_0^{\pi/2}\sin^2 x dx$ ------ (ii)

adding (i) and (ii), we get

$2I=\int_0^{\pi/2}(\cos^2 x+\sin^2 x)\,dx$

$=\int_0^{\pi/2}1.dx=[x]_0^{\pi/2}$

$\therefore 2I=\frac{\pi}{2} \Rightarrow I=\frac{\pi}{4}$

8. $I=\int_{2}^{3}\frac{x}{x^2+1}dx$

put $x^2+1=t \Rightarrow 2x\,dx=dt$

$\therefore x dx=\frac{1}{2}dt$

when x = 2 then t = 5 and

when x = 3 then t = 10

$I=\frac{1}{2}\int_{5}^{10}\frac{dt}{t}=\frac{1}{2}[\log t]_5^{10}$

$=\frac{1}{2}[\log 10-\log 5]$

$=\frac{1}{2}\log\frac{10}{5}=\frac{1}{2}\log 2$

9. $I=\int_0^1\frac{\tan^{-1}x}{1+x^2}dx$

put $\tan^{-1}x=t$

$\Rightarrow \frac{1}{1+x^2}dx=dt$

when x = 0 then t = 0 and

when x = 1 then $t=\frac{\pi}{4}$

$\therefore I=\int_0^{\pi/4}t\,dt=\left[\frac{t^2}{2}\right]_0^{\pi/4}$

$=\frac{\frac{\pi^2}{16}-0}{2}=\frac{\pi^2}{32}$

10. $I=\int_{-\pi/4}^{\pi/4}\sin^2x\,dx=2\int_0^{\pi/4}\sin^2x\,dx$

$=2\int_0^{\pi/4}\frac{1-\cos 2x}{2}dx=\int_0^{\pi/4}(1-\cos 2x)\,dx$

$=\left[x-\frac{\sin 2x}{2}\right]_0^{\pi/4}=\frac{\pi}{4}-\frac{1}{2}\sin\frac{\pi}{2}$

$=\frac{\pi}{4}-\frac{1}{2}$

11. $I=\int_0^1 x(1-x)^n dx=\int_0^1(1-x).x^n dx$

$=\int_0^1(x^n-x^{n+1})\,dx=\left[\frac{x^n}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1$

$=\frac{1}{n+1}-\frac{1}{n+2}=\frac{n+2-n-1}{(n+1)(n+2)}=\frac{1}{n^2+3n+2}$

12. $I=\int_{-1}^{1}e^x dx=[e^x]_{-1}^{1}$

$=e^1-e^{-1}=e-\frac{1}{e}$

INDEFINITE INTEGRATION (03 MARKS)

1. $I=\int(5x+3)\sqrt{2x-1}\,dx$

$=(5x+3)\int\sqrt{2x-1}\,dx-\int\left\{\frac{d}{dx}(5x+3)\int\sqrt{2x-1}\,dx\right\}dx$

$=(5x+3)\frac{(2x-1)^{3/2}}{\frac{3}{2}\times 2}-\int 5\left\{\frac{(2x-1)^{3/2}}{\frac{3}{2}.2}\right\}dx$

$=\frac{1}{3}(5x+3)(2x-1)^{3/2}-\frac{5}{3}\int(2x-1)^{3/2}dx$

$=\frac{1}{3}(5x+3)(2x-1)^{3/2}-\frac{5}{3}.\frac{(2x-1)^{5/2}}{\frac{5}{2}.2}+c$

$=\frac{1}{3}(5x+3)(2x-1)^{3/2}-\frac{1}{3}(2x-1)^{5/2}+c$

$=\frac{1}{3}(2x-1)^{3/2}[5x+3-(2x-1)]+c$

$=\frac{1}{3}(2x-1)^{3/2}(3x+4)+c$

2. $I=\int\frac{x(x+3)}{x+5}dx=\int\frac{x^2+3x}{x+5}dx$

$=\int\left[(x-2)+\frac{10}{x+5}\right]dx$

$=\frac{x^2}{2}-2x+10\log|x+5|+c$

3. $I=\int\sin^4x\,dx=\int(\sin^2x)^2dx$

$=\int\left(\frac{1-\cos 2x}{2}\right)^2dx$

$=\frac{1}{4}\int(1+\cos^2 2x-2\cos 2x)\,dx$

$=\frac{1}{4}\int\left(1+\frac{1+\cos 4x}{2}-2\cos 2x\right)dx$

$=\frac{1}{4}\int\frac{2+1+\cos 4x-4\cos 2x}{2}dx$

$=\frac{1}{8}\int(3+\cos 4x-4\cos 2x)\,dx$

$=\frac{3x}{8}+\frac{\sin 4x}{32}-\frac{\sin 2x}{4}+c$

4. $I=\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$

$=\int\frac{(2\cos^2x-1)-(2\cos^2\alpha-1)}{\cos x-\cos\alpha}dx$

$=2\int\frac{\cos^2x-\cos^2\alpha}{\cos x-\cos\alpha}dx$

$=2\int\frac{(\cos x+\cos\alpha)(\cos x-\cos\alpha)}{(\cos x-\cos\alpha)}dx$

$=2\int(\cos x+\cos\alpha)\,dx$

$=2\sin x+2x\cos\alpha+c$

5. $$I=\int \tan^4 x dx=\int (\tan^2 x \,.\, \tan^2 x)\, dx$$
$$=\int \tan^2 x(\sec^2 x-1)\, dx$$
$$=\int (\tan^2 x \sec^2 x-\tan^2 x)\, dx$$
$$=\int (\tan^2 x \sec^2 x)\, dx-\int \tan^2 x\, dx$$
$$=\int \tan^2 x \,.\, \sec^2 x dx-\int (\sec^2 x-1)\, dx$$
$$=\int \tan^2 x \,.\, \sec^2 x dx-\int \sec^2 x\, dx+\int 1\, dx$$
put $\tan x=t$ in 1st integral,
$$\Rightarrow \sec^2 x dx=dt$$
$$\therefore I=\int t^2 dt-\tan x+x$$
$$=\frac{t^3}{3}-\tan x+x+c$$
$$=\frac{\tan^3 x}{3}-\tan x+x+c$$

6. $$I=\int \tan^3 x\, dx=\int \tan x \tan^2 x\, dx$$
$$=\int \tan x(\sec^2 x-1)\, dx$$
$$\int \tan x \sec^2 x\, dx-\int \tan x\, dx$$
put $\tan x=t$ in first integral,
$$\Rightarrow \sec^2 x=\frac{dt}{dx}\Rightarrow \sec^2 x\, dx=dt$$
$$\therefore I=\int t dt-\log|\sec x|$$
$$=\frac{t^2}{2}-\log|\sec x|+c$$
$$=\frac{\tan^2 x}{2}-\log|\sec x|+c$$

7. $$I=\int \frac{dx}{x^2-a^2}=\int \frac{1}{(x+a)(x-a)}dx$$
$$=\frac{1}{2a}\int \frac{(x+a)-(x-a)}{(x+a)(x-a)}dx$$
$$=\frac{1}{2a}\int (\frac{1}{x-a}-\frac{1}{x+a})\, dx$$
$$=\frac{1}{2a}[\log|x-a|-\log|x+a|]+c$$
$$=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$$

8. $$I=\int \frac{dx}{a^2-x^2}=\int \frac{1}{(a+x)(a-x)}dx$$
$$=\frac{1}{2a}\int \frac{(a+x)+(a-x)}{(a+x)(a-x)}dx$$
$$=\frac{1}{2a}\int (\frac{1}{a-x}+\frac{1}{a+x})\, dx$$
$$=\frac{1}{2a}[-\log|a-x|+\log|a+x|]+c$$
$$=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$$

9. $$I=\int \frac{dx}{x^2+a^2}$$
put $x=a\tan\theta$ *then* $dx=a\sec^2\theta d\theta$
$$\therefore I=\int \frac{a\sec^2\theta\, d\theta}{a^2+a^2\tan^2\theta}=\int \frac{a\sec^2\theta}{a^2\sec^2\theta}d\theta$$
$$=\frac{1}{a}\int d\theta=\frac{1}{a}\theta+c=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$$

10. $$I=\int \frac{dx}{\sqrt{x^2-a^2}}$$
put $x=a\sec\theta$
$$\therefore dx=a\sec\theta.\tan\theta d\theta$$
$$\therefore I=\int \frac{a\sec\theta.\tan\theta d\theta}{\sqrt{a^2\sec^2\theta-a^2}}=\int \frac{a\sec\theta.\tan\theta}{a\tan\theta}d\theta$$
$$=\int \sec\theta d\theta \;=\log|\sec\theta+\tan\theta|+c_1$$
$$=\log\left|\frac{x}{a}+\sqrt{\frac{x^2}{a^2}-1}\right|+c_1$$
$$=\log|x+\sqrt{x^2-a^2}|-\log|a|+c_1$$
$$=\log|x+\sqrt{x^2-a^2}|+c,$$
where $c=c_1-\log|a|$

11. $$I=\int \frac{dx}{\sqrt{a^2-x^2}}$$
put $x=a\sin\theta$, then $dx=a\cos\theta d\theta$
$$\therefore I=\int \frac{a\cos\theta d\theta}{\sqrt{a^2-a^2\sin^2\theta}}=\int \frac{a\cos\theta}{a\cos\theta}d\theta$$
$$=\int d\theta \;=\theta+c=\sin^{-1}\frac{x}{a}+c$$

12. $$I=\int \frac{dx}{\sqrt{x^2+a^2}}$$
put $x=a\tan\theta$, then $dx=a\sec^2\theta d\theta$
$$\therefore I=\int \frac{a\sec^2\theta d\theta}{\sqrt{a^2\tan^2\theta+a^2}}=\int \frac{a\sec^2 d\theta}{a\sec\theta}$$
$$=\int \sec\theta d\theta \;=\log|\sec\theta+\tan\theta|+c_1$$
$$=\log\left|\frac{x}{a}+\sqrt{\frac{x^2}{a^2}+1}\right|+c_1$$
$$=\log|x+\sqrt{x^2+a^2}|-\log|a|+c_1$$
$$=\log|x+\sqrt{x^2+a^2}|+c$$
where $c=c_1-\log|a|$

13. $$I=\int \frac{dx}{x^2-6x+13}=\int \frac{dx}{(x-3)^2+4}$$
$$=\int \frac{dx}{(x-3)^2+(2)^2}$$
put $x-3=t$,
then $dx=dt$
$$I=\int \frac{dt}{(t)^2+(2)^2}=\frac{1}{2}\tan^{-1}\frac{t}{2}+c$$
$$=\frac{1}{2}\tan^{-1}\frac{x-3}{2}+c$$

14 $$I=\int \frac{x+3}{\sqrt{5-4x-x^2}}dx$$

$$let\ x+3=A.\frac{d}{dx}(5-4x-x^2)+B$$
$$=A(-4-2x)+B$$

equating the cofficients of x and the constant terms from both sides.

$$-2A=1 \quad \text{and} -4A+B=3$$
$$\Rightarrow A=-\frac{1}{2}, \text{and } B=1$$
$$\therefore I=-\frac{1}{2}\int \frac{-4-2x}{\sqrt{5-4x-x^2}}dx+\int \frac{1}{\sqrt{5-4x-x^2}}dx$$
$$=-\frac{1}{2}I_1+I_2 ------ (i)$$

In I_1, put $5-4x-x^2=t, \therefore (-4-2x)dx=dt$

$$\therefore I_1=\int \frac{dt}{\sqrt{t}}=2\sqrt{t}+c_1$$
$$=2\sqrt{5-4x-x^2}+c_1 --- (ii)$$

Now,

$$I_2=\int \frac{dx}{\sqrt{5-4x-x^2}}=\int \frac{dx}{\sqrt{9-(x+2)^2}}$$

put $x+2=t \Rightarrow dx=dt$

$$I_2=\int \frac{dt}{\sqrt{3^2-t^2}}=\sin^{-1}\frac{t}{3}+c_2$$
$$=\sin^{-1}\frac{x+2}{3}+c_2 ------ (iii)$$

Substituting the value of I_1 and I_2 from (ii) and (iii) in (i) we get,

$$I=-\frac{1}{2}(2\sqrt{5-4x-x^2})+\sin^{-1}\frac{x+2}{3}+c_2-\frac{c_1}{2}$$
$$=-\sqrt{5-4x-x^2}+\sin^{-1}\frac{x+2}{3}+c,$$

where $c=-\frac{1}{2}c_1+c_2$

15. $$I=\int \frac{x+2}{\sqrt{x^2-1}}dx$$
$$=\int (\frac{x}{\sqrt{x^2-1}}+\frac{2}{\sqrt{x^2-1}})dx$$
$$=\int \frac{x}{\sqrt{x^2-1}}dx+\int \frac{2}{\sqrt{x^2-1}}dx$$
$$=\int \frac{x}{\sqrt{x^2-1}}dx+2\log|x+\sqrt{x^2-1}|+c_1$$

Now,

Let $I_1=\int \frac{x}{\sqrt{x^2-1}}dx=\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx$

put $x^2-1=t \Rightarrow 2xdx=dt$

$$\therefore I_1=\frac{1}{2}\int \frac{dt}{\sqrt{t}}=\frac{1}{2}\times 2\sqrt{t}+c_2=\sqrt{x^2-1}+c_2$$
$$\therefore I=\sqrt{x^2-1}+2\log|x+\sqrt{x^2-1}|+c_1+c_2$$
$$=\sqrt{x^2-1}+2\log|x+\sqrt{x^2-1}|+c$$

where $c=c_1+c_2$

16. $$I=\int \frac{x}{(x-1)(x-2)(x-3)}dx$$

let $\frac{x}{(x-1)(x-2)(x-3)}$

$$=\frac{A}{(x-1)}+\frac{B}{(x-2)}+\frac{C}{(x-3)} -- (i)$$
$$=\frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$
$$\Rightarrow x=A(x-2)(x-3)+B(x-1)(x-3)$$
$$+C(x-1)(x-2) -- (ii)$$

put $x=1,2,3$ in (ii) successively we get,

$$1=A(-1)(-2)\Rightarrow A=\frac{1}{2}$$
$$2=B(1)(-1)\Rightarrow -B=2$$
$$3=C(2)(1) \quad \Rightarrow c=\frac{3}{2}$$
$$\therefore I=\int \left[\frac{1}{2(x-1)}-\frac{2}{x-2}+\frac{3}{2(x-3)}\right]dx$$
$$=\frac{1}{2}\int \frac{1}{x-1}dx-2\int \frac{1}{x-2}dx+\frac{3}{2}\int \frac{1}{x-3}dx$$
$$\therefore I=\frac{1}{2}\log|x-1|-2\log|x-2|+\frac{3}{2}\log|x-3|+c$$

17. $I=\int e^x \sin x dx$ .... (1)

Integrating by parts, we get

$$I=e^x\int \sin x\,dx-\int \left\{\frac{d}{dx}(e^x)\int \sin x\,dx\right\}dx$$
$$=e^x(-\cos x)-\int e^x(-\cos x)dx$$
$$=-e^x\cos x+\int e^x\cos x\,dx$$
$$=-e^x\cos x+e^x\int \cos x\,dx-\int \left\{\frac{d}{dx}(e^x)\int \cos x\,dx\right\}dx$$
$$=-e^x\cos x+e^x\sin x-\int e^x\sin x\,dx+c_1$$

or, $I=e^x(\sin x-\cos x)-I+c_1$; from (i)

or, $I+I=e^x(\sin x-\cos x)+c_1$

$$2I=e^x(\sin x-\cos x)+c_1$$
$$\therefore I=\frac{e^x}{2}(\sin x-\cos x)+\frac{c_1}{2}$$
$$\therefore \int e^x\sin x dx=\frac{e^x}{2}(\sin x-\cos x)+c;$$

where $c=\frac{c_1}{2}$

18. $I=\int x\sin^{-1}x\,dx$

put $x=\sin\theta \Rightarrow dx=\cos\theta d\theta$

$$\therefore I=\int \theta\sin\theta.\cos\theta d\theta=\frac{1}{2}\int \theta\sin 2\theta d\theta$$

Integrating by parts, we get;

$$I=\frac{1}{2}\left[\theta\int \sin 2\theta d\theta-\int \left\{\frac{d(\theta)}{d\theta}\int \sin 2\theta d\theta\right\}d\theta\right]$$
$$=\frac{1}{2}\left[\theta\left(\frac{-\cos 2\theta}{2}\right)-\int \frac{-\cos 2\theta}{2}d\theta\right]$$
$$=\frac{1}{4}\left[-\theta\cos 2\theta+\frac{\sin 2\theta}{2}\right]+c$$
$$=\frac{1}{4}\left[-\theta(1-2\sin^2\theta)+\frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}}\right]+c$$
$$=\frac{1}{4}[-\sin^{-1}x(1-2x^2)+x\sqrt{1-x^2}]+c$$
$$I=\frac{1}{4}[(2x^2-1)\sin^{-1}x+x\sqrt{1-x^2}]+c$$

19. $$I=\int \frac{(x^2+1)e^x}{(x+1)^2}dx$$
$$=\int \frac{x^2-1+2}{(x+1)^2}e^x dx$$
$$=\int e^x\left[\frac{x^2-1}{(x+1)^2}+\frac{2}{(x+1)^2}\right]dx$$
$$=\int e^x\left[\frac{x-1}{(x+1)}+\frac{2}{(x+1)^2}\right]dx$$
consider $f(x)=\frac{x-1}{x+1}$,

then $f'(x)=\frac{2}{(x+1)^2}$

Thus, the given integrand is of the form $e^x[f(x)+f'(x)]$

Hence, $\int \frac{x^2+1}{(x+1)^2}e^x dx=\frac{x-1}{x+1}.e^x+c$

20. $$I=\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$$
$$=\int 2\tan^{-1}x\,dx$$
Integrating by parts, we get
$$I=\tan^{-1}x\int 2dx-\int\left[\frac{d}{dx}(\tan^{-1}x)\int 2dx\right]dx$$
$$=2x\tan^{-1}x-\int \frac{2x}{1+x^2}dx+c$$
$$=2x\tan^{-1}x-\log|1+x^2|+c$$

21. $$I=\int \sqrt{x^2+2x+5}\,dx$$
$$=\int \sqrt{(x+1)^2+(2)^2}\,dx$$
$$=\frac{x+1}{2}\sqrt{(x+1)^2+(2)^2}+\frac{2^2}{2}\log\left|(x+1)+\sqrt{(x+1)^2+(2)^2}\right|+c$$
$$=\frac{x+1}{2}\sqrt{x^2+2x+5}+2\log\left|x+1+\sqrt{x^2+2x+5}\right|+c$$

22. $$I=\int \sqrt{1+3x-x^2}\,dx$$
$$=\int \sqrt{1-(x^2-3x)}\,dx$$
$$=\int \sqrt{1-\left\{(x)^2-2.\frac{3}{2}.x+\frac{9}{4}-\frac{9}{4}\right\}}\,dx$$
$$=\int \sqrt{1+\frac{9}{4}-(x-\frac{3}{2})^2}\,dx$$
$$=\int \sqrt{\frac{13}{4}-(x-\frac{3}{2})^2}\,dx$$
$$=\int \sqrt{(\frac{\sqrt{13}}{2})^2-(x-\frac{3}{2})^2}\,dx$$
$$=\left(\frac{x-\frac{3}{2}}{2}\right)\sqrt{(\frac{\sqrt{13}}{2})^2-(x-\frac{3}{2})^2}+\frac{(\frac{\sqrt{13}}{2})^2}{2}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right)+c$$
$$=\frac{2x-3}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+c$$

23. $$I=\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$$
put $\sin^{-1}x=t \Rightarrow \frac{1}{\sqrt{1-x^2}}dx=dt$
$$I=\int t\sin t\,dt$$
Integrating by parts, we get
$$I=t\int \sin t\,dt-\int\left\{\frac{d(t)}{dt}\int \sin t\,dt\right\}dt$$
$$=t(-\cos t)-\int 1.(-\cos t)\,dt$$
$$=-t\cos t+\sin t+c$$
$$=-\sin^{-1}x\sqrt{1-x^2}+x+c$$
$$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx=-\sin^{-1}x\sqrt{1-x^2}+x+c$$

DEFINITE INTEGRATION (03 MARKS)

1. we have,
$$I=\int_0^1 \frac{dx}{\sqrt{x+1}+\sqrt{x}}$$
$$=\int_0^1\left[\frac{\sqrt{x+1}-\sqrt{x}}{(\sqrt{x+1}+\sqrt{x})(\sqrt{x+1}-\sqrt{x})}\right]dx$$
$$=\int_0^1 \frac{\sqrt{x+1}-\sqrt{x}}{(x+1-x)}dx$$
$$=\int_0^1 (\sqrt{x+1}-\sqrt{x})\,dx$$
$$=\left[\frac{2}{3}(x+1)^{3/2}-\frac{2}{3}x^{3/2}\right]_0^1$$
$$=\frac{2}{3}[(x+1)^{3/2}-x^{3/2}]_0^1$$
$$=\frac{2}{3}[(2)^{3/2}-1^{3/2}-(1^{3/2}-0)]$$
$$=\frac{2}{3}(2\sqrt{2}-1-1)=\frac{2}{3}(2\sqrt{2}-2)$$
$$=\frac{4}{3}(\sqrt{2}-1)$$

2.
$$I=\int_0^{\pi/2}\frac{dx}{1+\sin x}\ =\int_0^{\pi/2}\frac{dx}{1+\cos(\frac{\pi}{2}-x)}$$
$$=\int_0^{\pi/2}\frac{dx}{2\cos^2(\frac{\pi}{4}-\frac{x}{2})}$$
$$=\frac{1}{2}\int_0^{\pi/2}\sec^2(\frac{\pi}{4}-\frac{x}{2})\,dx$$
$$=-\frac{1}{2}\left[\frac{\tan(\frac{\pi}{4}-\frac{x}{2})}{\frac{1}{2}}\right]_0^{\pi/2}$$
$$=-\left[\tan(\frac{\pi}{4}-\frac{x}{2})\right]_0^{\pi/2}$$
$$=-\left[\tan(\frac{\pi}{4}-\frac{\pi}{4})-\tan\frac{\pi}{4}\right]$$
$$=-[\tan 0-1]\ =-(0-1)=1$$

3.
$$I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx\ ---\ (i)$$
$$=\int_0^{\pi/2}\frac{\sqrt{\sin(\frac{\pi}{2}-x)}}{\sqrt{\sin(\frac{\pi}{2}-x)}+\sqrt{\cos(\frac{\pi}{2}-x)}}dx$$
$$\text{or, } I\ =\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx\ ----\ (ii)$$
Adding (i) and (ii) we get,
$$2I=\int_0^{\pi/2}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$
$$=\int_0^{\pi/2}1dx\ =[x]_0^{\pi/2}=\frac{\pi}{2}-0$$
$$=\frac{\pi}{2}$$
$$\therefore I=\frac{\pi}{4}\text{ Ans.}$$

4.
$$I=\int_0^4|x-1|dx$$
$$=\int_0^1|x-1|dx+\int_1^4|x-1|dx$$
$$=-\int_0^1(x-1)\,dx+\int_1^4(x-1)\,dx$$
$$=-\left[\frac{x^2}{2}-x\right]_0^1+\left[\frac{x^2}{2}-x\right]_1^4$$
$$=-\left[(\frac{1}{2}-1)-0\right]+\left[(\frac{16}{2}-4)-(\frac{1}{2}-1)\right]$$
$$=\frac{1}{2}+(4+\frac{1}{2})=5$$

5.
$$I=\int_0^{\pi/2}\log\sin x dx\ ---\ (i)$$
$$=\int_0^{\pi/2}\log\sin(\frac{\pi}{2}-x)\,dx$$
$$=\int_0^{\pi/2}\log\cos x dx\ ---\ (ii)$$
Adding (i) and (ii) we get,
$$2I=\int_0^{\pi/2}(\log\sin x+\log\cos x)\,dx$$
$$=\int_0^{\pi/2}\log(\sin x.\cos x)\,dx$$
$$=\int_0^{\pi/2}\log(\frac{\sin 2x}{2})\,dx$$
$$=\int_0^{\pi/2}\log\sin 2x\,dx-\int_0^{\pi/2}\log 2\,dx$$
put $2x=t$ in 1st integral

$\therefore\ dx=\frac{1}{2}dt$

when $x=0$ then $t=0$,

when $x=\frac{\pi}{2}$, then $t=\pi$
$$\therefore 2I=\frac{1}{2}\int_0^{\pi}\log\sin t\,dt-\log 2[x]_0^{\pi/2}$$
$$=\frac{2}{2}\int_0^{\pi/2}\log\sin t\,dt-\frac{\pi}{2}\log 2$$
$$=\int_0^{\pi/2}\log\sin x dx-\frac{\pi}{2}\log 2\ \text{ (by changing variable t to x)}$$
$$=I-\frac{\pi}{2}\log 2$$
$$\therefore 2I-I=-\frac{\pi}{2}\log 2$$
$$\therefore I=-\frac{\pi}{2}\log 2$$

6.
$$I=\int_0^{\pi/4}\log(1+\tan x)\,dx\ ---\ (i)$$
$$=\int_0^{\pi/4}\log(1+\tan(\frac{\pi}{4}-x))\,dx$$
$$=\int_0^{\pi/4}\log(1+\frac{1-\tan x}{1+\tan x})\,dx$$
$$=\int_0^{\pi/4}\log(\frac{1+\tan x+1-\tan x}{1+\tan x})\,dx$$
$$=\int_0^{\pi/4}\log(\frac{2}{1+\tan x})\,dx\ ---\ (ii)$$
Adding (i) and (ii) we get,
$$2I=\int_0^{\pi/4}\log(1+\tan x)\,dx$$
$$+\int_0^{\pi/4}\log(\frac{2}{1+\tan x})\,dx$$

$$= \int_0^{\pi/4} \log(1+\tan x)\,dx + \int_0^{\pi/4} [\log 2 - \log(1+\tan x)]dx$$

$$= \int_0^{\pi/4} \log(1+\tan x)\,dx + \int_0^{\pi/4} \log 2\,dx - \int_0^{\pi/4} \log(1+\tan x)\,dx$$

$$= \int_0^{\pi/4} \log 2 dx = \log 2 [x]_0^{\pi/4} = \frac{\pi}{4}\log 2$$

$$\therefore I = \frac{\pi}{8}\log 2$$

7. $$I = \int_0^{2\pi} |\cos x| dx$$

$$= \int_0^{\pi/2} |\cos x| dx + \int_{\pi/2}^{3\pi/2} |\cos x| dx + \int_{3\pi/2}^{2\pi} |\cos x| dx$$

$$= \int_0^{\pi/2} \cos x\,dx - \int_{\pi/2}^{3\pi/2} \cos x\,dx + \int_{3\pi/2}^{2\pi} \cos x\,dx$$

$$= [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{3\pi/2} + [\sin x]_{3\pi/2}^{2\pi}$$

$$= (\sin\frac{\pi}{2} - \sin 0) - (\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}) + (\sin 2\pi - \sin\frac{3\pi}{2})$$

$$= (1-0) - (-1-1) + [0-(-1)]$$

$$= 1+2+1 = 4$$

8. $$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$$

$$= \int_{\pi/6}^{\pi/3} \left(\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\right)dx \text{ ----- (i)}$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \text{ --- (ii)}$$

Adding (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} 1 dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

$$\therefore I = \frac{\pi}{12}$$

9. $$I = \int_0^{2\pi} \cos^5 x\,dx$$

$$= 2\int_0^{\pi} \cos^5 x\,dx \text{ ---- (i)}$$

$[\because \cos(2\pi - x) = \cos x]$

Let $I = \int_0^{\pi} \cos^5 x\,dx$ --- (ii)

$$= \int_0^{\pi} \cos^5(\pi - x)\,dx$$

$$= -\int_0^{\pi} \cos^5 x\,dx \text{ ---- (iii)}$$

$[\because \cos(\pi - x) = -\cos x]$

adding (ii) and (iii), we get,

$2I = 0$

$\therefore I = 0,$

Hence, $\int_0^{2\pi} \cos^5 x\,dx = 2I = 2\times 0 = 0$

10. $$I = \int_0^{\pi} \frac{x}{1+\sin x}dx \text{ ---- (i)}$$

$$= \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}dx$$

$$= \int_0^{\pi} \frac{\pi - x}{1+\sin x}dx \text{ ---- (ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{x+\pi-x}{1+\sin x}dx$$

$$= \pi\int_0^{\pi} \frac{1}{1+\sin x}dx$$

$$= \pi\int_0^{\pi} \frac{1-\sin x}{1-\sin^2 x}dx$$

$$= \pi\int_0^{\pi} \frac{1-\sin x}{\cos^2 x}dx$$

$$= \pi\int_0^{\pi} (\sec^2 x - \sec x.\tan x)\,dx$$

$$= \pi[\tan x - \sec x]_0^{\pi}$$

$$= \pi[(\tan\pi - \sec\pi) - (\tan 0 - \sec 0)]$$

$$= \pi[0-(-1)-(0-1)] = 2\pi$$

$$\therefore I = \pi$$

11. $$I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x}dx \text{ --- (i)}$$

$$= \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$= \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{\cos x + \sin x}dx \text{ --- (ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{x+\frac{\pi}{2}-x}{\sin x + \cos x}dx$$

$$= \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\sin x + \cos x}dx$$

$$\therefore I=\frac{\pi}{4}\int_0^{\pi/2}\frac{1}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)}dx$$
$$=\frac{\pi}{4\sqrt{2}}\int_0^{\pi/2}\frac{1}{\cos\left(x-\frac{\pi}{4}\right)}dx$$
$$=\frac{\pi}{4\sqrt{2}}\int_0^{\pi/2}\sec\left(x-\frac{\pi}{4}\right)dx$$
$$=\frac{\pi}{4\sqrt{2}}\left[\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right]_0^{\pi/2}$$
$$=\frac{\pi}{4\sqrt{2}}\left[\log\left|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right|-\log\left|\sec\frac{\pi}{4}-\tan\frac{\pi}{4}\right|\right]$$
$$=\frac{\pi}{4\sqrt{2}}[\log(\sqrt{2}+1)-\log(\sqrt{2}-1)]$$
$$=\frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)=\frac{\pi}{4\sqrt{2}}\log(\sqrt{2}+1)^2$$
$$\therefore I=\frac{\pi}{2\sqrt{2}}\log(\sqrt{2}+1)$$

12. $$I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}dx$$
$$=\int_{-a}^{a}\frac{a-x}{\sqrt{a^2-x^2}}dx$$
$$=\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}dx-\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx$$
Since, $\frac{x}{\sqrt{a^2-x^2}}$ is an odd function.

Hence $\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx=0$
$$\therefore I=\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}dx-0$$
$$=\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}dx$$
$$=a\int_{-a}^{a}\frac{1}{\sqrt{a^2-x^2}}dx$$
$$=2a\int_0^{a}\frac{dx}{\sqrt{a^2-x^2}}=2a\left[\sin^{-1}\frac{x}{a}\right]_0^a$$
$$=2a[\sin^{-1}1-\sin^{-1}0]=2a\left(\frac{\pi}{2}-0\right)$$
$$I=\cancel{2}a.\frac{\pi}{\cancel{2}}=\pi a$$

DEFINITE INTEGRATION
AS THE LIMIT OF SUM

1. We have $\int_a^b f(x)\,dx=\lim_{h\to 0}\sum_{r=1}^{n}hf(a+rh)$

where $nh=b-a$ and $n\to\infty$

Here $f(x)=x^2, a=0, b=1$

$$\therefore f(a+rh)=f(0+rh)=f(rh)$$
$$=(rh)^2=r^2h^2$$
and $nh=b-a=1-0=1$
$$\therefore \int_0^1 f(x)\,dx=\lim_{h\to 0}h\sum_{r=1}^{n}f(a+rh)$$
$$=\lim_{h\to 0}h\sum_{r=1}^{n}r^2h^2$$
$$=\lim_{h\to 0}h^3\sum_{r=1}^{n}r^2$$
$$=\lim_{h\to 0}\frac{h^3n(n+1)(2n+1)}{6}$$
$$=\lim_{h\to 0}\frac{nh(nh+h)(2nh+h)}{6}$$
$$=\lim_{h\to 0}\frac{1(1+h)(2+h)}{6}$$
$$=\frac{1\times1\times2}{6}=\frac{1}{3}$$
$$\therefore \int_0^1 x^2dx=\frac{1}{3}$$

2. From definition
$$\int_a^b f(x)\,dx=\lim_{h\to 0}\sum_{r=1}^{n}hf(a+rh)$$
where $nh=b-a$, $n\to\infty$

Here $f(x)=e^x, a=0. b=1$

$\therefore f(a+rh)=f(0+rh)=f(rh)=e^{rh}$

and $nh=b-a=1-0=1$

From (i)
$$\int_0^1 f(x)\,dx=\lim_{h\to 0}h\sum_{r=1}^{n}f(a+rh)$$
$$=\lim_{h\to 0}h\sum_{r=1}^{n}e^{rh}$$
$$=\lim_{h\to 0}h(e^h+e^{2h}+e^{3h}+e^{4h}+..........+e^{nh})$$
$$=\lim_{h\to 0}h\frac{e^h(1-e^{nh})}{1-e^h}$$
$$=\lim_{h\to 0}h\frac{e^h(1-e^1)}{1-e^h}$$
$$=\lim_{h\to 0}\frac{he^h}{-1+e^h}(e-1)$$
$$=\lim_{h\to 0}\frac{e^h(e-1)}{\left(\frac{e^h-1}{h}\right)}$$
$$=\frac{\lim_{h\to 0}e^h(e-1)}{\lim_{h\to 0}\frac{(e^h-1)}{h}}$$
$$=\frac{e^0(e-1)}{1}=(e-1)$$

3. From definition

$$\int_a^b f(x)\,dx == \lim_{h \to 0} \sum_{r=1}^{n} hf(a+rh)$$

$$\text{Where}, nh = b - a \text{ and } n \to \infty$$

$$\text{Here } f(x) = x, \quad a = 0, b = 3$$

$$\therefore f(a+rh) = f(0+rh) = f(rh) = rh$$

$$\text{and } nh = b - a = 3 - 0 = 3$$

$$\text{From (i)} \int_0^3 f(x)\,dx = \lim_{h \to 0} h\sum_{r=1}^{n} f(a+rh)$$

$$= \lim_{h \to 0} h\sum_{r=1}^{n} rh$$

$$= \lim_{h \to 0} h^2\sum_{r=1}^{n} r$$

$$= \lim_{h \to 0} h^2 \frac{n(n+1)}{2}$$

$$= \lim_{h \to 0} \frac{nh(nh+h)}{2}$$

$$= \lim_{h \to 0} \frac{3(3+h)}{2}$$

$$= \frac{9}{2} \quad \text{Ans..}$$

4. From definition

$$\int_a^b f(x)\,dx = \lim_{h \to 0} \sum_{r=1}^{n} hf(a+rh) -- (i)$$

$$\text{Where}, nh = b - a \text{ and } n \to \infty$$

$$\text{Here}, f(x) = x + e^{2x}, a = 0, b = 4$$

$$\therefore nh = b - a = 4 - 0 = 4$$

$$\therefore f(a+rh) = f(0+rh) = f(rh) = rh + e^{2rh}$$

$\therefore$ From (i),

$$\int_0^4 (x+e^{2x})dx = \lim_{h \to 0} h\sum_{r=1}^{n}(rh + e^{2rh})$$

$$= \lim_{h \to 0} h^2\sum_{r=1}^{n} r + \lim_{h \to 0} h\sum_{r=1}^{n} e^{2rh}$$

$$= \lim_{h \to 0} h^2 \frac{n(n+1)}{2} + \lim_{h \to 0} h \frac{e^{2h}(e^{2nh}-1)}{e^{2h}-1}$$

$$= \lim_{h \to 0} \frac{nh(nh+h)}{2} + \lim_{h \to 0} \frac{h\,e^{2h}(e^{2\times 4}-1)}{e^{2h}-1}$$

$$= \lim_{h \to 0} \frac{4(4+h)}{2} + \lim_{h \to 0} \frac{e^{2h}(e^8-1)}{\left(\frac{e^{2h}-1}{2h}\right).2}$$

$$= \frac{16}{2} + \frac{e^0(e^8-1)}{1\times 2}$$

$$= 8 + \frac{e^8-1}{2} = \frac{16+e^8-1}{2}$$

$$= \frac{e^8+15}{2} \quad \text{Ans..}$$