Chapter: 5 Continuity and Differentiability सांतत्य तथा अवकलनीयता

बहुविकल्पीय प्रश्न (MCQ)

1 Marks Question:-

1. **If $f(x) = x^2 + 2x$ then $f(2)$=?**
यदि $f(x) = x^2 + 2x$ तो $f(2)$=?
(a) 4 (b) 8
(c) 2 (d) 6

2. **A function $f(x) = x^2$ is continuous at**
(a) 2 (b) 3
(c) 0 (d) All of the above
फलन $f(x) = x^2$ संतत होता है
(a) 2 पर (b) 3 पर
(c) 0 पर (d) उपरोक्त सभी

3. **The function** $f(x) = \begin{cases} 2x - 1, & x<0 \\ 2x + 1, & x\geq 0 \end{cases}$ **is discontinuous at**
(a) 1 (b) 2
(c) 0 (d) None of these
फलन $f(x) = \begin{cases} 2x - 1, & x<0 \\ 2x + 1, & x\geq 0 \end{cases}$ **संतत नहीं है**
(a) 1 पर (b) 2 पर
(c) 0 पर (d) इनमें से कोई नहीं

4. **Function f(x) is continuous at x=a if**
फलन f(x), x=a पर संतत है यदि
(a) $\lim\limits_{x\to a^+} f(x) = \lim\limits_{x\to a^-} f(x) = f(a)$
(b) $\lim\limits_{x\to a^+} f(x) \neq \lim\limits_{x\to a^-} f(x) = f(a)$
(c) $\lim\limits_{x\to a^+} f(x) = \lim\limits_{x\to a^-} f(x) \neq f(a)$
(d) None of these इनमें से कोई नहीं

5. **प्रत्येक नियत फलन होता है।**
Every Constant function is
(a) Discontinuous (असंतत)
(b) Continuous (संतत)
(c) Discontinuous and Continuous (असंतत और संतत)
(d) None of these (इनमें से कोई नहीं)

6. **Which of the following function is continuous**
निम्नलिखित फलन मे से कौन सा संतत है–
(a) Identity function (तत्समक फलन)
(b) Polynomial function (बहुपद फलन)
(c) Rational function (परिमेय फलन)
(d) All of the above (उपरोक्त सभी)

7. **If f and g be continuous function then**
यदि f और g संतत फलन है, तो
(a) f+g is also continuous (f+g भी संतत हैं)
(b) f.g is also continuous (f.g भी संतत हैं)
(c) Both (a) and (b)/ दोनो (a) तथा (b)
(d) None of these./ इनमें से कोई नहीं।

8. **The value of k for which**
$f(x) = \begin{cases} \frac{\sin 5x}{3x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$
is continuous at x = 0
यदि x = 0 पर $f(x) = \begin{cases} \frac{\sin 5x}{3x}, & \text{यदि } x \neq 0 \\ k, & \text{यदि } x = 0 \end{cases}$
संतत है तो k का मान होगा–
(a) $\frac{1}{3}$ **(b) 0**
(c) $\frac{3}{5}$ **(d)** $\frac{5}{3}$

9. **Let** $f(x) = x^{3/2}$. **Then** $f'(o)$ = ?
माना $f(x) = x^{3/2}$. $f'(o)$ = ?
(a) $\frac{3}{2}$ (b) $\frac{1}{2}$
(c) does not exist (प्राप्त नहीं है) (d) None of these (इनमें से कोई नहीं)

10. **The function $f(x) = |x|, \forall\, x \in R$ is**
फलन $f(x) = |x|, \forall\, x \in R$ है–
(a) Continuous but not differentiable at x=0
x=o पर संतत परन्तु अवकलनीय नहीं
(b) differentiable but not continuous at x=0
x=0 पर अवकलनीय परन्तु संतत नहीं
(c) Neither continuous nor differentiable at x=0
x=0 पर, ना संतत ना अवकलनीय
(d) None of these . (इनमें से कोई नहीं)

11. **If the function** $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & x\neq \pi/2\\ 3, & x=\pi/2\end{cases}$ **be continuous at** $x=\pi/2$ **then the value of k is**

यदि $x=\pi/2$ पर $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & x\neq \pi/2\\ 3, & x=\pi/2\end{cases}$ संतत है तो k का मान है।

(a) -3 (b) -5
(c) 6 (d) 3

12. **If** $y=\sin(x^2+5)$ **then** $\frac{dy}{dx}=?$

यदि $y=\sin(x^2+5)$ तो $\frac{dy}{dx}=?$

(a) $2x.\sin(x^2+5)$ (b) $2x.\cos(x^2+5)$
(c) $x.\cos(x^2+5)$ (d) None

13. **If** $y=\cos(\sin x)$ **then** $\frac{dy}{dx}=?$

(a) $-\cos x.\sin(\sin x)$ (b) $\cos x.\sin(\sin x)$
(c) $-\sin(\sin x)$ (d) $-\cos(\sin x)$

14. **If** $y=x^2$ **then** $\frac{dy}{dx}=?$

(a) x (b) 2x
(c) -2 (d) -x

15. **if** $y=x^{-3}$ **then** $\frac{dy}{dx}=?$

(a) $-3x^{-4}$ (b) $3x^4$
(c) $3x^2$ (d) None

16. **If** $y=\cos(\sqrt{x})$ **then** $\frac{dy}{dx}=?$

(a) $\frac{\sin\sqrt{x}}{2\sqrt{x}}$ (b) $\frac{-\sin\sqrt{x}}{\sqrt{x}}$
(c) $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$ (d) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$

17. **If** $2x+3y=\sin x$ **then** $\frac{dy}{dx}=$?

(a) $\cos x$ (b) $-\cos x$
(c) $\frac{\cos x+2}{3}$ (d) $\frac{\cos x-2}{3}$

18. **If** $y=e^{x^3}$ **then** $\frac{dy}{dx}=?$

(a) $3x^2.e^{x^3}$ (b) $x^2.e^{x^3}$
(c) $3x^2.e^{x^2}$ (d) None

19. **If** $y=\sin x^3$ **then** $\frac{dy}{dx}=?$

(a) $\sin x^3$ (b) $-\cos x^3$
(c) $3x^2.\cos x^3$ (d) 0

20. **If** $y=\log\sin x$ **then** $\frac{dy}{dx}=?$

(a) $\cot x$ (b) $\sin x$
(c) $\tan x$ (d) $-\cot x$

21. **If** $y=3^x$ **then** $\frac{dy}{dx}=?$

(a) 3^x (b) $\log 3$
(c) $3^x.\log 3$ (d) None

22. **If** $y=\sin^{-1}(x^2)$ **then** $\frac{dy}{dx}=?$

(a) $\frac{2x}{\sqrt{x^4-1}}$ (b) $\frac{2x}{\sqrt{x^4+1}}$
(c) $\frac{x}{\sqrt{x^2-1}}$ (d) $\frac{1}{\sqrt{x^4-1}}$

23. **If** $y=\tan^{-1}(x^3)$ **then** $\frac{dy}{dx}=?$

(a) $\frac{3x^2}{x^2-1}$ (b) $\frac{x^2}{x^2+1}$
(c) $\frac{x^2}{x-1}$ (d) $\frac{3x^2}{x^6+1}$

24. **If** $y=\cos^{-1}x^3$ **then** $\frac{dy}{dx}=?$

(a) $\frac{-1}{1+x}$ (b) $\frac{2}{\sqrt{1+x}}$
(c) $\frac{-1}{2\sqrt{x}\,(1+x)}$ (d) None

25. **If** $y=\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ **then** $\frac{dy}{dx}=?$

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$
(c) 1 (d) -1

26. **If** $x=at^2$, $y=2at$ **then** $\frac{dy}{dx}=?$

(a) $\frac{1}{t}$ (b) $\frac{-1}{t^2}$
(c) $\frac{-2}{t}$ (d) None

27. **If** $x=a\sec\theta$, $y=b\tan\theta$ **then** $\frac{dy}{dx}=?$

(a) $\frac{b}{a}\sec\theta$ (b) $\frac{b}{a}\operatorname{cosec}\theta$
(c) $\frac{b}{a}\cot\theta$ (d) None

28. **if** $x=t$, $y=2t$ **then** $\frac{dy}{dx}=?$

(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) None

29. **If** $y=\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ **then** $\frac{dy}{dx}=?$

(a) 1 (b) -1
(c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

30. **If** $y=\sqrt{x^5}$ **then find** $\frac{dy}{dx}=?$

(a) $5x^4$ (b) $\frac{5x^4}{\sqrt{x^5}}$
(c) $\frac{5x^4}{2\sqrt{x^5}}$ (d) None

MCQ Answers
बहुविकल्पीय प्रश्न उत्तर

1. b	11. c	21. c
2. d	12. b	22. a
3. c	13. a	23. d
4. a	14. b	24. d
5. b	15. a	25. c
6. d	16. c	26. a
7. c	17. d	27. b
8. d	18. a	28. b
9. c	19. c	29. c
10. a	20. a	30. c

अति लघु उत्तरीय प्रश्न (Very Short Questions) (2 Marks)

1. **Prove that the function f(x) = 5x-3 is continuous at x=0.**

   सिद्ध करें कि x=0 पर फलन f(x) = 5x-3 संतत है।

2. **Show that the function $f(x) = x^2$ is differentiable at x = 1 and find f'(1)**

   सिद्ध करें x = 1 पर फलन $f(x) = x^2$ अवकलनीय है और f'(1) ज्ञात करें।

3. **If** $y = \sin 4x$ **then find** $\frac{dy}{dx}$.

   यदि $y = \sin 4x$ तो $\frac{dy}{dx}$ ज्ञात करें।

4. **If** $y = \cot\sqrt{x}$ **then find** $\frac{dy}{dx}$.

5. **If** $y = \sqrt{\sin x}$ **then find** $\frac{dy}{dx}$.

6. **If** $y = \sin 5x.\cos 3x$ **then find** $\frac{dy}{dx}$

7. **If** $y = \sqrt{x}$ **then find** $\frac{dy}{dx}$.

8. **If** $y = (2x + 3)^5$ **then find** $\frac{dy}{dx}$.

9. **If** $y = e^{x^2}$ **then find** $\frac{dy}{dx}$.

10. **If** $y = \log(2x + 1)$ **then find** $\frac{dy}{dx}$.

11. **If** $y = e^{3x}$ **then find** $\frac{dy}{dx}$.

12. **If** $y = \cos\frac{x}{2}$ **then find** $\frac{dy}{dx}$.

13. **If** $y = x.\sin x$ **then find** $\frac{dy}{dx}$.

14. **If** $y = \tan^{-1}x^2$ **then find** $\frac{dy}{dx}$.

15. **If** $y = \cos^{-1}2x$ **then find** $\frac{dy}{dx}$.

16. **If** $y = \log(2x + 3)$ **then find** $\frac{dy}{dx}$.

17. **If** $y = \text{cosec}(\sqrt{x})$ **then find** $\frac{dy}{dx}$.

18. **If** $y = e^{\sin x}$ **then find** $\frac{dy}{dx}$.

19. **If** $y = \sec^{-1}(x^2)$ **then find** $\frac{dy}{dx}$.

20. **If** $y = (3x^2 + 2x + 1)^2$ **then find** $\frac{dy}{dx}$.

Solutions of very short questions
अति लघु उत्तरीय प्रश्न का हल

1. Given function $f(x) = 5x - 3$
   at $x = 0$
   $f(0) = 5(0)-3 = -3$

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$$
$$= \lim_{h\to 0} 5(0+h) - 3$$
$$= \lim_{h\to 0} 5h - 3$$
$$= 5(0) - 3 = -3$$
$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$$
$$= \lim_{h\to 0} 5(0-h) - 3$$
$$= \lim_{h\to 0} -5h - 3$$
$$= -5(0) - 3$$
$$= -3$$

Clearly $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0)$

$\therefore$ f (x) is continuous at x = 0

x = 0 पर f (x) संतत है

2. Given function $f(x) = x^2$
   at $x = 1$

$$R\ f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h\to 0} \frac{(1+h)^2 - (1)^2}{h}$$
$$= \lim_{h\to 0} \frac{1 + 2h + h^2 - 1}{h}$$
$$= \lim_{h\to 0} \frac{2h + h^2}{h}$$
$$= \lim_{h\to 0} (2 + h)$$
$$= 2 + 0 = 2$$

$$\text{and } L\ f'(1) = \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h}$$
$$= \lim_{h\to 0} \frac{(1-h)^2 - (1)^2}{-h}$$
$$= \lim_{h\to 0} \frac{1-2h+h^2-1}{-h}$$
$$= \lim_{h\to 0} \frac{h^2-2h}{-h}$$
$$= \lim_{h\to 0} \frac{h(h-2)}{-h}$$
$$= 2$$

$\therefore$ R f' (1) = Lf'(1) = 2

$\therefore$ f(x) is differentiable at x = 1 and f'(1) = 2

3. Given $y = \sin 4x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \sin 4x}{dx}$$
$$= \cos 4x \times 4 \times (1)$$
$$= 4 \cos 4x$$

4. Given $y = \cot\sqrt{x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \cot\sqrt{x}}{dx}$$
$$= \frac{d \cot\sqrt{x}}{d\sqrt{x}} \times \frac{d\sqrt{x}}{dx}$$
$$= -\operatorname{cosec}^2\sqrt{x} \times \frac{1}{2\sqrt{x}}$$
$$= \frac{-\operatorname{cosec}^2\sqrt{x}}{2\sqrt{x}}$$

5. Given $y = \sqrt{\sin x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\sqrt{\sin x}}{dx}$$
$$= \frac{d\sqrt{\sin x}}{d \sin x} \times \frac{d \sin x}{dx}$$
$$= \frac{1}{2\sqrt{\sin x}} \times \cos x$$
$$= \frac{\cos x}{2\sqrt{\sin x}}$$

6. Given $y = \sin 5x \cdot \cos 3x$

$$y = \frac{1}{2} \cdot 2 \sin 5x \cdot \cos 3x$$
$$y = \frac{1}{2}\left[\sin(5x + 3x) + \sin(5x-3x)\right]$$
$$y = \frac{1}{2}\left[\sin 8x + \sin 2x\right]$$

d.w.r. to x

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{d \sin 8x}{dx} + \frac{d \sin 2x}{dx}\right]$$
$$\frac{dy}{dx} = \frac{1}{2}\left[\cos 8x \times 8 + \cos 2x \times 2\right]$$
$$\frac{dy}{dx} = 4 \cos 8x + \cos 2x$$

7. Given $y = \sqrt{x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\sqrt{x}}{dx}$$
$$= \frac{1}{2\sqrt{x}}$$

8. Given $y = (2x + 3)^5$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d (2x + 3)^5}{dx}$$
$$= \frac{d (2x + 3)^5}{d (2x + 3)} \times \frac{d (2x + 3)}{dx}$$
$$= 5(2x + 3)^4 \times [2 \cdot (1) + 0]$$
$$= 10(2x + 3)^4$$

9. Given $y = e^{x^2}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\, e^{x^2}}{dx}$$
$$= \frac{d\, e^{x^2}}{dx^2} \times \frac{dx^2}{dx}$$
$$= e^{x^2} \times 2x$$
$$= 2x \cdot e^{x^2}$$

10. Given $y = \log (2x + 1)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \log(2x+1)}{dx}$$
$$= \frac{d \log(2x+1)}{d(2x+1)} \times \frac{d(2x+1)}{dx}$$
$$= \frac{1}{(2x+1)} \times (2 + 0)$$
$$= \frac{2}{2x+1}$$

11. Given $y = e^{-3x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\, e^{-3x}}{dx}$$
$$= \frac{d\, e^{-3x}}{d(-3x)} \times \frac{d(-3x)}{dx}$$
$$= e^{-3x} \times (-3)$$
$$= -3 \cdot e^{-3x}$$

12. Given $y = \cos \frac{x}{2}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \cos \frac{x}{2}}{dx}$$
$$= \frac{d \cos x/2}{d x/2} \times \frac{d x/2}{dx}$$
$$= -\sin \frac{x}{2} \times \frac{1}{2}$$
$$= -\frac{1}{2} \sin \frac{x}{2}$$

13. Given $y = x.\sin x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\, x.\sin x}{dx}$$
$$= x \cdot \frac{d \sin x}{dx} + \sin x. \frac{d\, x}{dx}$$
$$= x \cdot (\cos x) + \sin x.(1)$$
$$= x \cdot \cos x + \sin x$$

14. Given $y = \tan^{-1} x^2$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \tan^{-1} x^2}{dx}$$

$$= \frac{d \tan^{-1} x^2}{dx^2} \times \frac{dx^2}{dx}$$

$$= \frac{1}{1+(x^2)^2} \times 2x$$

$$= \frac{2x}{1+x^4}$$

15. Given $y = \cos^{-1} 2x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \cos^{-1} 2x}{dx}$$

$$= \frac{d \cos^{-1} 2x}{d2x} \times \frac{d\, 2x}{dx}$$

$$= \frac{d \cos^{-1} 2x}{d2x} \times \frac{d\, 2x}{dx}$$

$$= \frac{-1}{\sqrt{1-(2x)^2}} \times 2$$

$$= \frac{-2}{\sqrt{1-4x^2}}$$

16. Given $y = \log(2x+3)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \log (2x+3)}{dx}$$

$$= \frac{d \log (2x+3)}{d(2x+3)} \times \frac{d\,(2x+3)}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2x+3} \times 2$$

$$= \frac{2}{2x+3}$$

17. Given $y = \text{cosec}\,(\sqrt{x})$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\, \text{cosec}\,(\sqrt{x})}{dx}$$

$$= \frac{-\text{cosec}\sqrt{x} \cdot \cot\sqrt{x}}{2\sqrt{x}}$$

18. Given $y = e^{\sin x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\, e^{\sin x}}{dx}$$

$$= e^{\sin x} \cdot \cos x$$

19. Given $y = \sec^{-1}(x^2)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \sec^{-1}(x^2)}{dx}$$

$$= \frac{1}{x^2 \cdot \sqrt{x^4-1}} \times 2x$$

$$= \frac{2}{x\sqrt{x^4-1}}$$

20. Given $y = (3x^2 + 2x + 1)^2$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\,(3x^2 + 2x + 1)^2}{dx}$$

$$= 2\,(3x^2 + 2x + 1) \cdot (6x + 2)$$

$$= 4\,(3x + 1) \cdot (3x^2 + 2x + 1)$$

Short Question (लघु उत्तरीय प्रश्न) (3 Marks)

1. **Discuss the continuity of the function f(x) at x = 0 if**

$$f(x) = \begin{cases} 2x - 1, & x<0 \\ 2x + 1, & x \geq 0 \end{cases}$$

2. **Find the value of k for which**

$$f(x) = \begin{cases} kx + 5, & x \leq 2 \\ x-1, & x > 2 \end{cases}$$

is continuous at x = 2

3. **Find the value of k for which**

$$f(x) = \begin{cases} \dfrac{1 - \cos 4x}{8x^2}, & x \neq 2 \\ k, & x = 0 \end{cases}$$

is continuous at x = 2,

यदि $f(x) = \begin{cases} \dfrac{1 - \cos 4x}{8x^2}, & x \neq 2 \\ k, & x = 0 \end{cases}$

x = 2 पर संतत है तो **k** का मान ज्ञात करें।

4. **Show that the function** $f(x) = \begin{cases} 1+x, & x \leq 2 \\ 5-x, & x > 2 \end{cases}$ **is not differentiable at** x = 2

सिद्ध करें कि फलन $f(x) = \begin{cases} 1+x, & x \leq 2 \\ 5-x, & x > 2 \end{cases}$ x = 2 पर अवकलनीय नहीं है।

Find $\frac{dy}{dx}$ ($\frac{dy}{dx}$ ज्ञात करें)

5. $y = \cot^3 x^2$

6. $y = \sqrt{\sin x^3}$

7. $y = \sin(\log x)$

8. $y = \sqrt{e^{\sqrt{x}}}$

9. $y = \cos^{-1}(\cot x)$

10. $y = \sin(\tan^{-1} x)$

11. $y = \log(\tan^{-1} x)$

12. $x^2 + y^2 = 4$

13. $x = at^2$, $y = 2at$

14. $y = x^{10} + 2x^5 + \sqrt{\sin x}$

15. $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

16. $y = \cos^{-1}\sqrt{1-x^2}$

17. $y = x^x$

18. $y = b \sin\theta$, $x = a\cos\theta$

19. $y = \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

20. $y = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

Solutions of short questions (लघु उत्तरीय प्रश्न का हल)

1. Given $f(x) = \begin{cases} 2x - 1, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$

at $x = 0$ ($x = 0$ पर)

$f(0) = 2\cdot(0)+1 = 1$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{h\to 0} f(0+h)$

$= \lim\limits_{h\to 0} 2(0+h) + 1$

$= 1$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} f(0-h)$

$= \lim\limits_{h\to 0} 2(0-h) + (-1)$

$= -1$

Clearly $f(0) = \lim\limits_{x\to 0^+} f(x) \neq \lim\limits_{x\to 0^-} f(x)$

$\therefore$ f(x) is discontinuous at $x = 0$

$x = 0$ पर f(x) असंतत है।

2. **Given,** $f(x) = \begin{cases} kx + 5, & x \leq 2 \\ x - 1, & x > 2 \end{cases}$

f(x) is continuous at $x = 2$

$x = 2$ पर f(x) संतत है

$\therefore f(2) = \lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^-} f(x)$

$\Rightarrow k\cdot(2) + 5 = \lim\limits_{h\to 0} f(2+h) = \lim\limits_{h\to 0} f(2-h)$

$\Rightarrow 2k + 5 = \lim\limits_{h\to 0} (2+h) - 1 = \lim\limits_{h\to 0} k(2-h) + 5$

$\Rightarrow 2k + 5 = \lim\limits_{h\to 0} (h + 1)$

$\Rightarrow 2k + 5 = 1$

$\Rightarrow 2k = 1-5$

$\Rightarrow 2k = -4$

$\Rightarrow k = -2$

3. **Given,** $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$

f(x) is continuous at $x = 0$

$x = 0$ पर f(x) संतत है।

$\therefore f(0) = \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^-} f(x)$

$\Rightarrow k = \lim\limits_{h\to 0} f(0+h)$

$\Rightarrow k = \lim\limits_{h\to 0} \frac{1 - \cos(4(0+h))}{8(0+h)^2}$

$\Rightarrow k = \lim\limits_{h\to 0} \frac{1 - \cos 4h}{8h^2}$

$\Rightarrow k = \lim\limits_{h\to 0} \frac{2\sin^2 2h}{8h^2}$

$\Rightarrow k = \lim\limits_{h\to 0} \frac{\sin^2 2h}{4h^2}$

$\Rightarrow k = \lim\limits_{h\to 0} \left(\frac{\sin 2h}{2h}\right)^2$

$\Rightarrow k = (1)^2$

$\Rightarrow k = 1$

4. **Given,**

$f(x) = \begin{cases} 1 + x, & x \leq 2 \\ 5 - x, & x > 2 \end{cases}$

at $x = 2$ ($x=2$ पर)

$Rf'(2) = \lim\limits_{h\to 0} \frac{f(2+h) - f(2)}{h}$

$= \lim\limits_{h\to 0} \frac{[5-(2+h)] - [1+2]}{h}$

$= \lim\limits_{h\to 0} \frac{5-2-h-3}{h}$

$= \lim\limits_{h\to 0} \frac{-h}{h}$

$= -1$

$Lf'(2) = \lim\limits_{h\to 0} \frac{f(2-h) - f(2)}{-h}$

$$= \lim_{h\to 0} \frac{[1+(2-h)] - [1+2]}{-h}$$

$$= \lim_{h\to 0} \frac{1+2-h-3}{-h}$$

$$= \lim_{h\to 0} \frac{-h}{-h}$$

$$= 1$$

Clearly $Rf'(2) \neq Lf'(2)$

$\therefore$ f(x) is not differentiable at x = 2

x = 2 **पर** f(x) **अवकलनीय नहीं है।**

5. **Given,**

$y = \cot^3 x^2 = (\cot x^2)^3$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d(\cot x^2)^3}{dx}$$

$$= \frac{d(\cot x^2)^3}{d(\cot x^2)} \times \frac{d \cot x^2}{dx^2} \times \frac{dx^2}{dx}$$

$$= 3(\cot x^2)^2 \times (-\text{cosec}^2 x^2) \times (2x)$$

$$= -6x \cdot \cot^2 x^2 \cdot \text{cosec}^2 x^2$$

6. Given

$y = \sqrt{\sin x^3}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\sqrt{\sin x^3}}{dx}$$

$$= \frac{d\sqrt{\sin x^3}}{d \sin x^3} \times \frac{d \sin x^3}{dx^3} \times \frac{dx^3}{dx}$$

$$= \frac{1}{2\sqrt{\sin x^3}} \times \cos x^3 \times 3x^2$$

$$= \frac{3x^2 \cdot \cos x^3}{2\sqrt{\sin x^3}}$$

7. Given

$y = \sin(\log x)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \sin(\log x)}{dx}$$

$$= \frac{d \sin(\log x)}{d(\log x)} \times \frac{d \log x}{dx}$$

$$= \cos(\log x) \times \frac{1}{x}$$

$$= \frac{\cos(\log x)}{x}$$

8. Given

$y = \sqrt{e^{\sqrt{x}}}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\sqrt{e^{\sqrt{x}}}}{dx}$$

$$= \frac{d\sqrt{e^{\sqrt{x}}}}{de^{\sqrt{x}}} \times \frac{de^{\sqrt{x}}}{d\sqrt{x}} \times \frac{d\sqrt{x}}{dx}$$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}} \times e^{\sqrt{x}} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{e^{\sqrt{x}}}{4\sqrt{x} \cdot \sqrt{e^{\sqrt{x}}}}$$

$$= \frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$$

9. Given

$y = \cos^{-1}(\cot x)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \cos^{-1}(\cot x)}{dx}$$

$$= \frac{d \cos^{-1}(\cot x)}{d \cot x} \times \frac{d \cot x}{dx}$$

$$= \frac{-1}{\sqrt{1-\cot^2 x}} \times (-\text{cosec}^2 x)$$

$$= \frac{\text{cosec}^2 x}{\sqrt{1-\cot^2 x}}$$

10. Given

$y = \sin(\tan^{-1} x)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \sin(\tan^{-1} x)}{dx}$$

$$= \frac{d \sin(\tan^{-1} x)}{d \tan^{-1} x} \times \frac{d \tan^{-1} x}{dx}$$

$$= \cos(\tan^{-1} x) \times \frac{1}{1+x^2}$$

$$= \frac{\cos(\tan^{-1} x)}{1+x^2}$$

11. Given $y = \log(\tan^{-1} x)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \log(\tan^{-1} x)}{dx}$$

$$= \frac{d \log(\tan^{-1} x)}{d \tan^{-1} x} \times \frac{d \tan^{-1} x}{dx}$$

$$= \frac{1}{\tan^{-1} x} \times \frac{1}{1+x^2} = \frac{1}{(1+x^2)\tan^{-1} x}$$

12. Given $x^2+y^2 = 4$

d.w.r. to x

$$2x + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \quad 2y \frac{dy}{dx} = -2x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{y}$$

13. Given

$x = at^2$ d.w.r. to t $x = 2at$ d.w.r. to t

$$\frac{dx}{dt} = 2at \ , \quad \frac{dy}{dt} = 2a$$

$$\text{Now} \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{2a}{2at} = \frac{1}{t}$$

14. Given $y = x^{10} + 2x^5 + \sqrt{\sin x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\,x^{10}}{dx} + \frac{d\,2x^5}{dx} + \frac{d\sqrt{\sin x}}{dx}$$

$$= 10\,x^9 + 10\,x^4 + \frac{d\sqrt{\sin x}}{d\sin x} \times \frac{d\sin x}{dx}$$

$$= 10\,x^9 + 10\,x^4 + \frac{1}{2\sqrt{\sin x}} \times \cos x$$

$$= 10\,x^9 + 10\,x^4 + \frac{\cos x}{2\sqrt{\sin x}}$$

15. Given

$$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Put $x = \tan\theta$ $\quad \therefore \ \theta = \tan^{-1} x$

$$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$y = \tan^{-1}(\tan 2\theta)$$

$$y = 2\theta = 2\tan^{-1}x$$

d.w.r. to x

$$\frac{dy}{dx} = \frac{2}{1+x^2}$$

16. Given

$$y = \cos^{-1}\sqrt{1-x^2}$$

Put $x = \sin\theta$ $\quad \therefore \ \theta = \sin^{-1} x$

$$y = \cos^{-1}\sqrt{1-\sin^2\theta}$$

$$y = \cos^{-1}\sqrt{\cos^2\theta}$$

$$y = \cos^{-1}(\cos\theta)$$

$$y = \theta$$

$$y = \text{Sin}^{-1}x$$

d.w.r. to x

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

17.

Given

$$y = x^x$$

taking log both sides

दोनों पक्षों में log लेने पर

$$\log y = \log x^x$$

$$\log y = x.\log x$$

d.w.r. to x

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$$

$$\frac{dy}{dx} = y[1+\log x]$$

$$= x^x(1+\log x)$$

18. Given

$x = a\cos\theta$ d.w.r. to θ $y = b\sin\theta$ d.w.r. to θ

$$\frac{dx}{d\theta} = -a\sin\theta, \ \frac{dy}{d\theta} = b\cos\theta$$

$$\therefore \ \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{b\cos\theta}{-a\sin\theta}$$

$$= \frac{-b}{a} \cdot \cot\theta$$

19. Given

$$y = \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow y = \frac{x}{2}$$

d.w.r. to x

$$\frac{dy}{dx} = \frac{1}{2}$$

20. Given

$$y = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$$

put $x = \sin\theta \quad \therefore \quad \theta = \sin^{-1} x$

$$y = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$

$$y = \sec^{-1}\left(\frac{1}{\sqrt{\cos^2\theta}}\right)$$

$$y = \sec^{-1}\left(\frac{1}{\cos\theta}\right)$$

$$y = \sec^{-1}(\sec\theta)$$

$$y = \theta$$

$$y = \sin^{-1} x$$

d.w.r. to x

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

लघु उत्तरीय प्रश्न (Short Question)

3 Marks Question:-

Find $\frac{dy}{dx}$ $\left(\frac{dy}{dx}\right.$ ज्ञात करें।$\left.\right)$

1. $2x + 3y = \sin x$

2. $xy + y^2 = \tan x + y$

3. $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

4. $y = \sin^{-1}(2x.\sqrt{1-x^2})$

5. $y = \sin(\tan^{-1} e^{-x})$

6. $y = \log(\cos e^x)$

7. $y = e^x + e^{x^2} + \ldots\ldots\ldots\ldots + e^{x^5}$

8. $y = (\log x)^{\cos x}$

9. $x^y + y^x = 1$

10. $y^x = x^y$

11. $(\cos y)^x = (\cos x)^y$

12. $x = a(\theta - \sin\theta),\ y = a(1 + \cos\theta)$

13. if $y = 5\cos x - 3\sin x$, then prove that $\frac{d^2y}{dx^2} + y = 0$

यदि $y = 5\cos x - 3\sin x$, तो सिद्ध करें $\frac{d^2y}{dx^2} + y = 0$

14. If $y = 3\sin x + \log x$ then find $\frac{d^2y}{dx^2}$

यदि $y = 3\sin x + \log x$ तो $\frac{d^2y}{dx^2}$ ज्ञात करें।

15. If $y = \tan^{-1}x$ then find $\frac{dy}{dx}$ at $x = 1$

यदि $y = \tan^{-1}x$ तो $x = 1$ पर $\frac{dy}{dx}$ ज्ञात करें।

16. Show that $f(x) = |x-2|$ is continuous but not differentiable at $x = 2$

सिद्ध कीजिए कि $x = 2$ पर $f(x) = |x-2|$ संतत है, परन्तु अवकलनीय नहीं है।

Solutions of short questions
(लघु उत्तरीय प्रश्न का हल)

1. $2x + 3y = \sin x$

diff . both sides w.r.t x

$$2 + 3\frac{dy}{dx} = \cos x$$

$$3\frac{dy}{dx} = \cos x - 2$$

$$\frac{dy}{dx} = \frac{\cos x - 2}{3}$$

2. $xy + y^2 = \tan x + y$

diff . both sides w.r.t x

$$x.\frac{dy}{dx} + y.1 + 2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}.(x + 2y - 1) = \sec^2 x - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec^2 x - y}{x + 2y - 1}$$

3. $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

put $x = \tan\theta \quad \therefore \quad \theta = \tan^{-1} x$

$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$

$y = \tan^{-1}(\tan 3\theta)$

$y = 3\theta$

$y = 3.\tan^{-1}x$

diff. both sides w.r. to x

$$\frac{dy}{dx} = \frac{3}{1+x^2}$$

4.

$y = \sin^{-1}(2x\sqrt{1-x^2})$

put $x = \sin\theta \quad \therefore \quad \theta = \sin^{-1} x$

$y = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$y = \sin^{-1}(2\sin\theta.\cos\theta)$

$y = \sin^{-1}(\sin 2\theta)$

$y = 2\theta$

$y = 2\sin^{-1}x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

5. $y = \sin(\tan^{-1} e^{-x})$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\sin(\tan^{-1} e^{-x})}{dx}$$

$$= \frac{d\sin(\tan^{-1} e^{-x})}{d(\tan^{-1} e^{-x})} \times \frac{d\tan^{-1} e^{-x}}{d e^{-x}} \times \frac{d e^{-x}}{d(x)}$$

$$= \cos(\tan^{-1} e^{-x}) \times \frac{1}{1+(e^{-x})^2} \times e^{-x} \times (-1)$$

$$= \frac{-e^{-x}.\cos(\tan^{-1} e^{-x})}{1+e^{-2x}}$$

6. $y = \log(\cos e^x)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\log(\cos e^x)}{dx}$$

$$= \frac{d\log(\cos e^x)}{d(\cos e^x)} \times \frac{d\cos e^x}{de^x} \times \frac{de^x}{dx}$$

$$= \frac{1}{\cos e^x} \times (-\sin e^x) \times e^x$$

$$= -e^x.\tan(e^x)$$

7. $y = e^x + e^{x^2} + \ldots + e^{x^5}$

$y = e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$

d.w.r. to x

$$\frac{dy}{dx} = e^x + 2x.e^{x^2} + 3x^2.e^{x^3} + 4x^3e^{x^4} + 5x^4.e^{x^5}$$

8. $y = (\log x)^{\cos x}$

taking log both sides

$\log y = \log(\log x)^{\cos x}$

$\log y = \cos x.\log(\log x)$

diff.both sides w.r.t x

$$\frac{1}{y}.\frac{dy}{dx} = \cos x.\frac{d\log(\log x)}{dx} + \log(\log x).\frac{d\cos x}{dx}$$

$$\Rightarrow \frac{dy}{dx} = y.\left[\cos x.\frac{1}{\log x} \times \frac{1}{x} + \log(\log x).(-\sin x)\right]$$

$$= (\log x)^{\cos x}.\left[\frac{\cos x}{x.\log x} - \sin x.\log(\log x)\right]$$

9.

$x^y + y^x = 1$

d.w.r to x both sides

$$\frac{d\,x^y}{dx} + \frac{d\,y^x}{dx} = \frac{d(1)}{dx}$$

$$\Rightarrow x^y.\left[\frac{y}{x}.\frac{dx}{dx} + \log x.\frac{dy}{dx}\right] + y^x\left[\frac{x}{y}.\frac{dy}{dx} + \log y.\frac{dx}{dx}\right] = 0$$

$$\Rightarrow \frac{y.x^y}{x} + x^y.\log x.\frac{dy}{dx} + \frac{x.y^x}{y}.\frac{dy}{dx} + y^x\log y = 0$$

$$\Rightarrow \frac{dy}{dx}\left[x^y.\log x + \frac{x.y^x}{y}\right] = -\left[y^x.\log y + \frac{y.x^y}{x}\right]$$

$$\Rightarrow \frac{dy}{dx}\left[x^y.\log x + x.y^{x-1}\right] = -\left[y^x.\log y + y.x^{y-1}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{-[y^x.\log y + yx^{y-1}]}{[x^y.\log x + xy^{x-1}]}$$

10.

$y^x = x^y$

taking log both sides

$\Rightarrow \log y^x = \log x^y$

$\Rightarrow x.\log y = y.\log x$

diff. both sides w.r.t x

$$\Rightarrow x.\frac{1}{y}.\frac{dy}{dx} + \log y = y.\frac{1}{x} + \log x.\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{x}{y} - \log x\right) = \frac{y}{x} - \log y$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(\frac{y}{x} - \log y\right)}{\left(\frac{x}{y} - \log x\right)} = \frac{y}{x}\left(\frac{y - x\log y}{x - y\log x}\right)$$

11. $(\cos x)^y = (\cos y)^x$

taking log both sides

दोनों पक्षों में log लेने पर

$y.\log(\cos x) = x.\log(\cos y)$

diff. both sides w.r.t x

$$y.\frac{1}{\cos x}(-\sin x) + \log(\cos x)\frac{dy}{dx} = x.\left(\frac{-\sin y}{\cos y}\right)\frac{dy}{dx} + \log(\cos y)$$

$$\Rightarrow -y\tan x + \log(\cos x)\frac{dy}{dx} = -x\tan y\frac{dy}{dx} + \log(\cos y)$$

$$\Rightarrow \frac{dy}{dx}[\log(\cos x) + x\tan y] = \log(\cos y) + y\tan x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\log(\cos y) + y\tan x}{\log(\cos x) + x\tan y}$$

12. $x = a(\theta - \sin\theta)$ $\quad y = a(1 + \cos\theta)$

d.w.r. to θ $\quad$ d.w.r. to θ

$$\frac{dx}{d\theta} = a(1 - \cos\theta), \quad \frac{dy}{d\theta} = -a\sin\theta$$

$$\text{Now } \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)}$$

$$= \frac{-2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$

$$= -\cot\frac{\theta}{2}$$

13.

$y = 5\cos x - 3\sin x$ —— (1)

d.w.r. to x

$$\frac{dy}{dx} = -5\sin x - 3\cos x \quad \text{—— (2)}$$

again d.w.r. to x

$$\frac{d^2y}{dx^2} = -5\cos x + 3\sin x \quad \text{—— (3)}$$

Adding (1) and (3) we get

(1) तथा (3) को जोड़ने पर

$$\frac{d^2y}{dx^2} + y = 0$$

14. $y = 3\sin x + \log x$ —— (1)

d.w.r. to x

$$\frac{dy}{dx} = 3\cos x + \frac{1}{x}$$

$$\frac{d^2y}{dx^2} = -3\sin x - \frac{1}{x^2}$$

15. $y = \tan^{-1}x$ —— (1)

d.w.r. to x

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

at x = 1 (x = 1 पर)

$$\left.\frac{dy}{dx}\right]_{x=1} = \frac{1}{1+(1)^2}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

16. Given

$f(x) = |x-2|$ —— (1)

at x = 2

$$\lim_{x\to2^+} f(x) = \lim_{h\to0} f(2+h) = \lim_{h\to0} |2+h-2|$$

$$= \lim_{h\to0} h = 0$$

$$\lim_{x\to2^-} f(x) = \lim_{h\to0} f(2-h) = \lim_{h\to0} |2-h-2|$$

$$= \lim_{h\to0} h = 0$$

and

$f(2) = |2-2| = 0$

$\therefore$ $f(2) = \lim_{x\to2^+} f(x) = \lim_{x\to2^-} f(x)$

$\therefore$ f(x) is continuous at x = 2 **Proved**

x = 2 पर f(x) संतत है।

Again :-

$$Rf'(2) = \lim_{h\to0}\frac{f(2+h) - f(2)}{h} = \lim_{h\to0}\frac{|2+h-2| - |2-2|}{h}$$

$$= \lim_{h\to0}\frac{h-0}{h} = 1$$

$$Lf'(2) = \lim_{h\to0}\frac{f(2-h) - f(2)}{-h} = \lim_{h\to0}\frac{|2-h-2| - |2-2|}{-h}$$

$$= \lim_{h\to0}\frac{h}{-h} = -1$$

$\therefore$ $Rf'(2) \neq Lf'(2)$

$\therefore$ f(x) is not differentiable at x = 2

x = 2 पर f(x) अवकलनीय है। **Proved**