Chapter: 10	Vector Algebra सदिश बीजगणित

बहुविकल्पीय प्रश्न (MCQ)

1 Marks Question:-

1. Position vector of the point (x,y,z) is

बिन्दु (x,y,z)का स्थिति सदिश है।

(A) $x\hat{i}-y\hat{j}-z\hat{k}$ (B) $x\hat{i}+y\hat{j}-z\hat{k}$

(C) $x\hat{i}-y\hat{j}+z\hat{k}$ (D) $x\hat{i}+y\hat{j}+z\hat{k}$

2. Position vector of the point (1,0,2) is

बिन्दु (1,0,2) का स्थिति सदिश है।

(A) $\hat{i}+\hat{j}+2\hat{k}$ (B) $\hat{i}+2\hat{j}$

(C) $\hat{i}+3\hat{k}$ (D) $\hat{i}+2\hat{k}$

3. Find the maginitude of vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$

सदिश $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ का परिमाण होगा।

(A) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ **(B)** $\sqrt{3}$

(C) 3 **(D)** $\frac{1}{\sqrt{3}}$

4. Magnitude of vector $2\hat{i}-7\hat{j}-3\hat{k}$ is

सदिश $2\hat{i}-7\hat{j}-3\hat{k}$ का परिमाण है।

(A) $\sqrt{61}$ **(B)** $\sqrt{62}$

(C) $\sqrt{64}$ **(D)** $\sqrt{32}$

5. If vector $2\hat{i}+3\hat{j}=x\hat{i}+y\hat{j}$ then x = ___

यदि सदिश $2\hat{i}+3\hat{j}=x\hat{i}+y\hat{j}$ हो तो x = ___

(A) 2 (B) 3

(C) 4 (D) 9

6. The projection of vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ is

सदिश $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ का सदिश $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ पर प्रक्षेप होगा।

(A) $\frac{10}{\sqrt{6}}$ **(B)** $\frac{15}{\sqrt{6}}$

(C) $\frac{5\sqrt{6}}{3}$ **(D)** $\frac{6\sqrt{5}}{3}$

7. The projection of vector $\hat{i}+\hat{j}$ on $\hat{i}-\hat{j}$ is

सदिश $\hat{i}+\hat{j}$ का सदिश $\hat{i}-\hat{j}$ पर प्रक्षेप होगा।

(A) 2 **(B) 0**

(C) -1 **(D) 1**

8. If $\vec{a}=5\hat{i}+7\hat{J}-3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}-\hat{k}$ then $\vec{a}-\vec{b}$ is

यदि $\vec{a}=5\hat{i}+7\hat{J}-3\hat{k}$ और $\vec{b}=2\hat{i}-3\hat{j}-\hat{k}$ तो $\vec{a}-\vec{b}$ होगा।

(A) $7\hat{i}+10\hat{j}-4\hat{k}$ **(B)** $3\hat{i}+10\hat{j}-2\hat{k}$

(C) $7\hat{i}+4\hat{j}-2\hat{k}$ **(D)** $10\hat{i}-21\hat{j}+3\hat{k}$

9. If $\vec{A}=2\hat{i}-3\hat{j}$ and $\vec{B}=-\hat{i}-\hat{j}$ then $\vec{A}+\vec{B}$ is equal to

यदि $\vec{A}=2\hat{i}-3\hat{j}$ और $\vec{B}=-\hat{i}-\hat{j}$ है तो $\vec{A}+\vec{B}$ = ?

(A) $3\hat{i}-4\hat{j}$ **(B)** $3\hat{i}-2\hat{j}$

(C) $\hat{i}-4\hat{j}$ **(D)** $2\hat{i}-3\hat{j}$

10. Find the unit vector in the direction of $\overrightarrow{AB}$ where $A(1,2,3)$ and $B(4,5,6)$ are the given point

सदिश $\overrightarrow{AB}$ के अनुदिश मात्रक सदिश ज्ञात कीजिए जहाँ $A(1,2,3)$ और $B(4,5,6)$ है।

(A) $5i+7\hat{j}+9\hat{k}$ **(B)** $3\hat{i}+3\hat{j}+3\hat{k}$

(C) $4\hat{i}+10\hat{J}+18\hat{K}$ **(D)** $-3\hat{i}+3\hat{j}-3\hat{k}$

11. If $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=4\hat{i}-2\hat{j}+2\hat{k}$ are perpendicular then $\lambda=?$

यदि $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ और $\vec{b}=4\hat{i}-2\hat{j}+2\hat{k}$ लम्बवत्त हो, तो $\lambda=?$

(A) 5 (B) 6

(C) 4 (D) -5

12. The scalar product of $5\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}-4\hat{j}+7\hat{k}$ is

सदिश $5\hat{i}+\hat{j}-3\hat{k}$ और $3\hat{i}-4\hat{j}+7\hat{k}$ का सदिश गुणक होगा।

(A) 15 **(B) -15**

(C) 10 **(D) -10**

13. Which vector direction is along the vector $\hat{i}+2\hat{j}+3\hat{k}$?

सदिश $\hat{i}+2\hat{j}+3\hat{k}$ के अनुदिश सदिश कौन है?

(A) $2\hat{i}-4\hat{j}+6\hat{k}$ **(B)** $\hat{i}-2\hat{j}+3\hat{k}$

(C) $2\hat{i}+4\hat{j}+6\hat{k}$ **(D)** $3(\hat{i}-2\hat{j}-2\hat{k})$

14. **If $\vec{a}$ and $\vec{b}$ are non.zero vector and $\vec{a}.\vec{b}=0$ then**

यदि $\vec{a}$ और $\vec{b}$ अशून्य सदिश और $\vec{a}.\vec{b}=0$ है तो

(A) $\vec{a}=\vec{b}$ **(B)** $\vec{a}\|\vec{b}$

(C) $|\vec{a}|=|\vec{b}|$ **(D)** $\vec{a}\perp\vec{b}$

15. **The Addative inverse of vector $-2\hat{i}+\hat{j}-\hat{k}$ is**

सदिश $-2\hat{i}+\hat{j}-\hat{k}$ का योज्य प्रतिलोम (ऋणात्मक) मान है।

(A) $-2\hat{i}-\hat{j}-\hat{k}$ **(B)** $\frac{\hat{i}}{2}+\hat{j}-\hat{k}$

(C) $\frac{1}{6}(-2\hat{i}+6-\hat{k})$ **(D)** $2\hat{i}-\hat{j}+\hat{k}$

16. **If $|\vec{a}|=1,\vec{b}|=2$ and $\vec{a}.\vec{b}=1$ then angle between $\vec{a}$ and $\vec{b}$ is**

यदि $|\vec{a}|=1,\vec{b}|=2$ और $\vec{a}.\vec{b}=1$ हो तो $\vec{a}$ और $\vec{b}$ के बीच का कोण होगा।

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ **(D)** $\frac{2\pi}{3}$

17. **The unit vector along the vector**

$\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$ **is**

सदिश $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$ के अनुदिश मात्रक सदिश होगा।

(A) $\frac{1}{\sqrt{5}}(2\hat{i}+3\hat{j}+\hat{k})$ **(B)** $\frac{1}{\sqrt{6}}(2\hat{i}+3\hat{j}+\hat{k})$

(C) $\frac{1}{\sqrt{14}}(2\hat{i}+3\hat{j}+\hat{k})$ **(D)** $\frac{1}{14}(2\hat{i}+3\hat{j}+\hat{k})$

18. **If $|\vec{a}|=3,|\vec{b}|=\frac{\sqrt{2}}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector then angle between $\vec{a}$ and $\vec{b}$ is**

यदि $|\vec{a}|=3,|\vec{b}|=\frac{\sqrt{2}}{3}$ और $\vec{a}\times\vec{b}$ एक ईकाई सदिश हो तो सदिश $\vec{a}$ और $\vec{b}$ के बीच का कोण होगा।

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ **(D)** $\frac{\pi}{2}$

19. **If $|\vec{a}|\neq 0,|\vec{b}|\neq 0$ and $\vec{a}\times\vec{b}=0$ then**

यदि $|\vec{a}|\neq 0,|\vec{b}|\neq 0$ और $\vec{a}\times\vec{b}=0$ हो तो

(A) $\vec{a}=\vec{b}$ **(B)** $\vec{a}\perp\vec{b}$

(C) $\vec{a}\parallel\vec{b}$ **(D)** $\vec{a}.\vec{b}=\vec{a}\times\vec{b}$

20. **If $|\vec{a}|=2,|\vec{b}|=3$ and $\vec{a}.\vec{b}=4$ then $|\vec{a}-\vec{b}|=?$**

यदि $|\vec{a}|=2,|\vec{b}|=3$ और $\vec{a}.\vec{b}=4$ है तो $|\vec{a}-\vec{b}|=?$

(A) $\sqrt{5}$ **(B)** 5

(C) $\frac{1}{5}$ **(D)** $\frac{1}{\sqrt{5}}$

21. **If θ is the angle between $\vec{a}$ and $\vec{b}$ such that $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$ then θ is**

यदि सदिश $\vec{a}$ और $\vec{b}$ के बीच का कोण θ है और जहाँ $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$ हो तो θ का मान होगा।

(A) 0 **(B)** $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ **(D)** π

22. **The direction cosine of the vector $\hat{i}+2\hat{j}+3\hat{k}$ is**

सदिश $\hat{i}+2\hat{j}+3\hat{k}$ का दिक् –कोसाइन होगा।

(A) 1,2,3 **(B)** $\frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$

(C) $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$ **(D)** $\frac{-1}{\sqrt{14}},\frac{-2}{\sqrt{14}},\frac{-3}{\sqrt{14}}$

VERY SHORT QUESTIONS

1. **Find the value of x, y and z, where vector $\vec{a}=x\hat{i}+2\hat{j}+z\hat{k}$ and $\vec{b}=2\hat{i}+y\hat{j}+\hat{k}$ are equal.**

x,y और z का मान ज्ञात कीजिए, जहाँ सदिश $\vec{a}=x\hat{i}+2\hat{j}+z\hat{k}$ और $\vec{b}=2\hat{i}+y\hat{j}+\hat{k}$ बराबर है।

2. **Find a vector in the direction of the vector $5\hat{i}-\hat{j}+2\hat{k}$. which has magnitude 8 units.**

सदिश $5\hat{i}-\hat{j}+2\hat{k}$ के अनुदिश सदिश ज्ञात कीजिए , जिसका परिमाण 8 इकाई है।

3. **If $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$ then find a unit vector in the direction of $(\vec{a}+\vec{b})$.**

यदि $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ और $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$ हो तो सदिश $(\vec{a}+\vec{b})$ के अनुदिश इकाई सदिश ज्ञात कीजिए।

4. **If A(1,2,-3) and B(-1,2,1) are two given point in the space then find $\overrightarrow{AB}$.**

यदि बिन्दु A(1,2,-3) और बिन्दु B(-1,2,1) अंतरिक्ष पर स्थित हो तो सदिश $\overrightarrow{AB}$ ज्ञात करें।

5. **Find the position vector of a point R which divides the line joining point P(1,2,-1) and Q(-1,1,1) in the ratio 2:1 internally.**

बिन्दु P(1,2,-1) और Q(-1,1,1) को मिलाने वाली रेखा को बिन्दु R, 2:1 के अनुपात में अन्तः विभाजित

करती हो तो बिन्दु R का बिन्दु सदिश क्या होगा ?

6. Find the value of P for which $P(\hat{i}+\hat{j}+\hat{k})$ is a unit vector.

P का मान ज्ञात करे जहाँ $P(\hat{i}+\hat{j}+\hat{k})$ एक इकाई सदिश है।

7. Find the angle between the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$

सदिश $\hat{i}-2\hat{j}+3\hat{k}$ और $3\hat{i}-2\hat{j}+\hat{k}$ के बीच का कोण ज्ञात कीजिए।

8. If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$ then show that vector $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular.

यदि $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ और $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$ तो सिद्ध करें कि सदिश $\vec{a}+\vec{b}$ और $\vec{a}-\vec{b}$ लम्बवत् है।

9. If $\vec{a}$ is unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=12$ then Find the value of $|\vec{x}|$

यदि $\vec{a}$ इकाई सदिश और $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=12$ हो तो $|\vec{x}|$ का मान ज्ञात कीजिए।

10. If $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$ then Find the value of $|\vec{a}\times\vec{b}|$

यदि $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ और $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$ हो तो $|\vec{a}\times\vec{b}|$ ज्ञात कीजिए।

11. If $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$ then Find the value of λ and μ.

यदि $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$ तो λ और μ को ज्ञात कीजिए।

12. Find the area of triangle whose vertices are A(1,1,2), B(2,3,5) and C(1,5,5).

एक त्रिभुज का क्षेत्रफल ज्ञात कीजिए, जिसके शीर्ष A(1,1,2), B(2,3,5) और C(1,5,5) हैं।

13. Show that the points $A(-2\hat{i}+3\hat{j}+5\hat{k}), B(\hat{i}+2\hat{j}+3\hat{k})$ and $C(7\hat{i}-\hat{k})$ are collinear.

दर्शाइए कि बिन्दु $A(-2\hat{i}+3\hat{j}+5\hat{k}), B(\hat{i}+2\hat{j}+3\hat{k})$ और $C(7\hat{i}-\hat{k})$ संरेख है।

14. Find the area of parallelogram whose adjcent sides are represented by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$

एक समांतर चतुर्भुज का क्षेत्रफल ज्ञात कीजिए जिसकी संलग्न भुजाएँ $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ और $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$ द्वारा निर्धारित है।

15. Find the projection of $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ on $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$

सदिश $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ का सदिश $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$ पर प्रक्षेप ज्ञात करे।

16. Find value (मान ज्ञात करें।)

$\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$

3 Marks Question (Vector)

1. Show that the point A, B and C whose position vectors $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$ respectively, are vertices of right angle triangle.

दर्शाइए कि बिन्दु A, B और C जिनके स्थिति सदिश क्रमशः $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}$ और $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$ है, एक समकोण त्रिभुज के शीर्षो का निर्माण करता है।

2. If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ such that $(\vec{a}+\lambda\vec{b})\perp\vec{c}$ then Find the value of λ.

यदि $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ और $\vec{c}=3\hat{i}+\hat{j}$ इस प्रकार है कि $(\vec{a}+\lambda\vec{b})\perp\vec{c}$ तो λ का मान ज्ञात कीजिए।

3. If A(1,2,3), B(-1,0,0) and C(0,1,2) be the vertices of ΔABC then find the value $\angle$ABC.

यदि किसी ΔABC के शीर्ष A,B,C क्रमशः (1,2,3), (-1,0,0) और (0,1,2) है तो $\angle$ABC का मान ज्ञात कीजिए।

4. If $\vec{a},\vec{b},\vec{c}$ be three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ Find the angle between $\vec{a}$ and $\vec{b}$.

यदि $\vec{a},\vec{b},\vec{c}$ तीन सदिश इस प्रकार है कि $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ और $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ है तो सदिश $\vec{a}$ और $\vec{b}$ के बीच कोण का मान ज्ञात करें।

5. If $\vec{a},\vec{b},\vec{c}$ are vectors such that $|\vec{a}|=5, |\vec{b}|=4, |\vec{c}|=3$ and each vector is perpendicular to the sum of the other two, then find $|\vec{a}+\vec{b}+\vec{c}|$.

यदि $\vec{a},\vec{b},\vec{c}$ सदिश इस प्रकार है कि $|\vec{a}|=5, |\vec{b}|=4, |\vec{c}|=3$ और प्रत्येक सदिश अन्य दो सदिशो के योग पर लम्बवत् है तो $|\vec{a}+\vec{b}+\vec{c}|$ का मान ज्ञात करें।

6. Show that the points A(1,2,7), B(2,6,3) and C(3,10,-1) are collinear.

दर्शाइए की बिन्दु A(1,2,7), B(2,6,3) और C(3,10,-1) संरेख है।

(MCQ) Answer, Chapter - Vector

1. D	9. C	17. C
2. B	10. B	18. B
3. B	11. A	19. C
4. B	12. D	20. A
5. A	13. C	21. B
6. A	14. D	22. C
7. B	15. D	
8. B	16. A	

Very Short Questions Answer

1. Given that $\vec{a}=\vec{b}$

$$\Rightarrow (x\hat{i}+2\hat{j}+z\hat{k})=(2\hat{i}+y\hat{j}+\hat{k})$$

so that, the component of $\hat{i},\hat{j}$ and $\hat{k}$ are equal.

$\therefore x=2, y=2, z=1$

2. *Let* $\vec{a}=5\hat{i}-\hat{j}+2\hat{k}$

Then $|\vec{a}|=\sqrt{5^2+(-1)^2+2^2}$

$=\sqrt{25+1+4}$

$=\sqrt{30}$

$\therefore$ *unit vector,*

$$\hat{a}=\frac{\vec{a}}{|\vec{a}|}=\left\{\frac{5}{\sqrt{30}}\hat{i}-\frac{1}{\sqrt{30}}\hat{j}+\frac{2}{\sqrt{30}}\hat{k}\right\}$$

Hence, the required vector $=8\hat{a}$

$$=8\left\{\frac{5}{\sqrt{30}}\hat{i}-\frac{1}{\sqrt{30}}\hat{j}+\frac{2}{\sqrt{30}}\hat{k}\right\}$$

3. *Given that*

$\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$

$b=-\hat{i}+\hat{j}-\hat{k}$

$\therefore \vec{a}+\vec{b}=(2-1)\hat{i}+(-1+1)\hat{j}+(2-1)\hat{k}$

$i+0\hat{j}+\hat{k}=\hat{i}+\hat{k}$

$|\vec{a}+\vec{b}|=\sqrt{1^2+1^2}=\sqrt{2}$

Unit vector along $(\vec{a}+\vec{b})$

$$=\frac{(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|}=\frac{1}{2}(\hat{i}+\hat{k})$$

4. Let O be the origin of the space.

then $\overline{OA}=\hat{i}+2\hat{j}-3\hat{k}$

& $\overline{OB}=-\hat{i}+2\hat{j}+\hat{k}$

$\therefore \overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$

$=(-\hat{i}+2\hat{j}+\hat{k})-(\hat{i}+2\hat{j}-3\hat{k})$

$=(-1-1)\hat{i}+(2\hat{j}-2\hat{j})+(1+3)\hat{k}$

$=-2\hat{i}+0\hat{j}+4\hat{k}=-2\hat{i}+4\hat{k}$

5. **Let O be the origin**

so that $\overrightarrow{OP}=\hat{i}+2\hat{j}-\hat{k}=\vec{a}$(Let)

$\overrightarrow{OQ}=-\hat{i}+\hat{j}+\hat{k}=\vec{b}$(Let)

Point R divides PQ internally in the ratio 2:1

then $\overrightarrow{OR}=\dfrac{m\vec{b}+n\vec{a}}{m+n}$ $[\because m:n=2:1]$

$$=\frac{2(-\hat{i}+\hat{j}+\hat{k})+1(\hat{i}+2\hat{j}-\hat{k})}{2+1}$$

$$=\frac{(-2+1)\hat{i}+(2+2)\hat{j}+(2-1)\hat{k}}{3}$$

$$=\frac{-\hat{i}+4\hat{j}+\hat{k}}{3}$$

Hence, the position vector of R is $\dfrac{-\hat{i}+4\hat{j}+\hat{k}}{3}$.

6. *Let* $\vec{a}=P(\hat{i}+\hat{j}+\hat{k})$

$|\vec{a}|=\sqrt{p^2+p^2+p^2}=\pm P\sqrt{3}$

We have given that $|\vec{a}|=1$

$\Rightarrow \pm P\sqrt{3}=1$

$P=\pm\dfrac{1}{\sqrt{3}}$

7.

Let $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$

$\vec{b}=3\hat{i}-2\hat{j}+\hat{k}$

Now $|\vec{a}|=\sqrt{1^2+(-2)^2+3^2}=\sqrt{1+4+9}=\sqrt{14}$

$|\vec{b}|=\sqrt{3^2+(-2)^2+1^2}=\sqrt{9+4+1}=\sqrt{14}$

$\vec{a}.\vec{b}=(\hat{i}-2\hat{j}+3\hat{k}).(3\hat{i}-2\hat{j}+\hat{k})$

$=3+4+3=10$

Let θ *be the angle between* $\vec{a}$ *and* $\vec{b}$,

then $\cos\theta=\dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}=\dfrac{10}{\sqrt{14}.\sqrt{14}}=\dfrac{\cancel{10}^{5}}{\cancel{14}_{7}}$

$\cos\theta=\dfrac{5}{7}$

$\therefore \theta=\cos^{-1}\dfrac{5}{7}$

8.

Given that

$$\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$$
$$\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$$
$$Now\ \vec{a} + \vec{b} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$
$$\vec{a} - \vec{b} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$
$$\therefore (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = (6\hat{i} + 2\hat{j} - 8\hat{k}).(4\hat{i} - 4\hat{j} + 2\hat{k})$$
$$= 24 - 8 - 16$$
$$= 0$$

Hence, $(\vec{a} + \vec{b})$ *and* $(\vec{a} - \vec{b})$ *is perpendicular to each other.*

9.

$$(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$$
$$\Rightarrow \vec{x}.\vec{x} + \vec{x}.\vec{a} - \vec{a}.\vec{x} - \vec{a}.\vec{a} = 12$$
$$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$$
$$\Rightarrow |\vec{x}|^2 - 1^2 = 12 \left[\begin{array}{l} \because \vec{a} \text{ is unit vector} \\ \therefore |\vec{a}| = 1 \end{array}\right]$$
$$|\vec{x}|^2 = 13$$
$$\therefore |\vec{x}| = \sqrt{13}$$

10.

Given that $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$

$$\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$$
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$
$$= (-2 - 15)\hat{i} - (-4 - 9)\hat{j} + (10 - 3)\hat{k}$$
$$= -17\hat{i} + 13\hat{j} + 7\hat{k}$$
$$|\vec{a} \times \vec{b}| = \sqrt{(-17)^2 + (13)^2 + 7^2}$$
$$= \sqrt{289 + 169 + 49}$$
$$= \sqrt{507}$$

11.

Given that

$$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$$
$$Now \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = (2\lambda - 6)\hat{i} + (27 - 2\mu)\hat{j} + (2\lambda - 6)\hat{k}$$
$$\because (2\lambda - 6)\hat{i} + (27 - 2\mu)\hat{j} + (2\lambda - 6)\hat{k} = \vec{0}$$

Equating the coefficient of $\hat{i}, \hat{j}$ & $\hat{k}$*. We, get*

$$2\lambda - 6 = 0 \Rightarrow \lambda = 3$$
$$27 - 2\mu = 0 \Rightarrow \mu = \frac{27}{2}$$

12.

Let O be the origin

then $\overrightarrow{OA} = \hat{i} + \hat{j} + 2\hat{k}$

$$\overrightarrow{OB} = 2\hat{i} + 3\hat{j} + 5\hat{k}$$
$$\overrightarrow{OC} = \hat{i} + 5\hat{j} + 5\hat{k}$$
$$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$$
$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 0\hat{i} + 4\hat{j} + 3\hat{k}$$
$$\therefore \text{area of } \triangle ABC = \left|\frac{1}{2}(\overrightarrow{AB} \times \overrightarrow{AC})\right|$$
$$Now\ \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$
$$= (6 - 12)\hat{i} + (0 - 3)\hat{j} + (4 - 0)\hat{k}$$
$$= -6\hat{i} - 3\hat{j} + 4\hat{k}$$
$$Now\ \frac{1}{2}(\overrightarrow{AB} \times \overrightarrow{AC}) = 2\hat{i} - \frac{3}{2}\hat{j} - 3\hat{k}$$
$$\text{Area of } \triangle ABC = \left|\frac{1}{2}(\overrightarrow{AB} \times \overrightarrow{AC})\right| = \sqrt{9 + \frac{9}{4} + 4}$$
$$= \sqrt{\frac{61}{4}} = \frac{1}{2}\sqrt{61}\ Sq.Unit$$

13.

Let O be the origin. then

$$\overrightarrow{OA} = -2\hat{i} + 3\hat{j} + 5\hat{k}$$
$$\overrightarrow{OB} = \hat{i} + 2\hat{j} + 3\hat{k}$$
$$\overrightarrow{OC} = 7\hat{i} - \hat{k}$$
$$Now\ \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (-2\hat{i} + 3\hat{j} + 5\hat{k})$$
$$= (1 + 2)\hat{i} + (2 - 3)\hat{j} + (3 - 5)\hat{k}$$
$$= 3i - \hat{j} - 2\hat{k}$$
$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$$
$$= (7i - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$
$$= 6\hat{i} - 2\hat{j} - 4\hat{k}$$
$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$$
$$= (7\hat{i} - \hat{k}) - (-2\hat{i} + 3\hat{j} + 5\hat{k})$$
$$= 9\hat{i} - 3\hat{j} - 6\hat{k}$$
$$Now\ \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -2 \\ 9 & -3 & -6 \end{vmatrix}$$
$$= (6 - 6)\hat{i} + (-18 + 18)\hat{j} + (-9 + 9)\hat{k}$$
$$= 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

Hence, point A, B and C are collinears.

14.

Given that adjacent sides of Parallelogram are

$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

Then the area of parallelogram $= |\vec{a} \times \vec{b}|$

Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$

$= (-1 + 21)\hat{i} + (6 - 1)\hat{j} + (-7 + 3)\hat{k}$

$= 20\hat{i} + 5\hat{j} - 4\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{(20)^2 + 5^2 + (-4)^2}$

$= \sqrt{400 + 25 + 16}$

$= \sqrt{441} = 21$

Hence, area of parallelogram is 21 Sq. Unit.

15. *Projection of* $\vec{a}$ *on* $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$= \dfrac{(2\hat{i} - \hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{(1)^2 + (-2)^2 + 1^2}}$

$= \dfrac{2 + 2 + 1}{\sqrt{1 + 4 + 1}} = \dfrac{5}{\sqrt{6}} = \dfrac{5\sqrt{6}}{6}$

16. $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{i} \times \hat{k}) + \hat{k}.(i \times \hat{j})$

$= \hat{i}.\hat{i} + \hat{j}.(-\hat{j}) + \hat{k}.\hat{k}$ $\quad \left[\begin{array}{l} \because \hat{i} \times \hat{j} = \hat{k} \\ \hat{j} \times \hat{k} = \hat{i} \\ \hat{i} \times \hat{k} = -\hat{j} \end{array}\right]$

$= 1 - 1 + 1$

$= 1$

3 Marks (Vector) Questions Answer

1. *Let O be the origin.*

Then $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (2\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k})$

$= -\hat{i} + 3\hat{j} + 5\hat{k}$

$|\overrightarrow{AB}| = \sqrt{1 + 9 + 25} = \sqrt{35}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$

$= -\hat{i} - 2\hat{j} - 6\hat{k}$

$|\overrightarrow{BC}| = \sqrt{1 + 4 + 36} = \sqrt{41}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k})$

$= -2\hat{i} + \hat{j} - \hat{k}$

$|\overrightarrow{AC}| = \sqrt{4 + 1 + 1} = \sqrt{6}$

Now $|\overrightarrow{BC}|^2 = |\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2$

Hence the point A, B and C

are the vertices of the right angle triangle.

2.

Given that

$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{c} = 3\hat{i} + \hat{j}$

$\because (\vec{a} + \lambda\vec{b}) \perp \vec{c} \Leftrightarrow (\vec{a} + \lambda\vec{b}).\vec{c} = 0$

$\Rightarrow [(2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k})].(3\hat{i} + \hat{j}) = 0$

$\Rightarrow [(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}].(3\hat{i} + \hat{j}) = 0$

$\Rightarrow 3(2 - \lambda) + 2 + 2\lambda = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$

$\Rightarrow 8 - \lambda = 0$

$\therefore \lambda = 8$

3. *Let O be the origin. then*

$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{OB} = -\hat{i}$

$\overrightarrow{OC} = \hat{j} + 2\hat{k}$

$\because \angle ABC$ *makes between* $\overrightarrow{AB}$ & $\overrightarrow{BC}$

$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= -\hat{i} - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= -2\hat{i} - 2\hat{j} - 3\hat{k}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \hat{j} + 2\hat{k} + \hat{i}$

$= \hat{i} + \hat{j} + 2\hat{k}$

$\therefore |\overrightarrow{AB}.\overrightarrow{BC}| = |-2 - 2 - 6| = 10$

$|\overrightarrow{AB}| = \sqrt{4 + 4 + 9} = \sqrt{17}$

$|\overrightarrow{BC}| = \sqrt{1 + 1 + 4} = \sqrt{6}$

$\therefore \cos(\angle ABC) = \dfrac{\overrightarrow{AB}.\overrightarrow{BC}}{|\overrightarrow{AB}||\overrightarrow{BC}|}$

$= \dfrac{10}{\sqrt{17}.\sqrt{6}}$

$\therefore \angle ABC = \cos^{-1}\left(\dfrac{10}{\sqrt{102}}\right)$

4.

Let θ *be the angle between* $\vec{a}$ *and* $\vec{b}$

Now

$\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow (\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = (-\vec{c}).(-\vec{c})$

$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}=\vec{c}.\vec{c}$

$\Rightarrow |\vec{a}|^2+2\vec{a}.\vec{b}+|\vec{b}|^2=|\vec{c}|^2$

$[\because \vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta]$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta=|\vec{c}|^2$

$\Rightarrow 3^2+5^2+2.3.5\cos\theta=7^2$

$\Rightarrow 9+25+30\cos\theta=49$

$\Rightarrow 30\cos\theta=49-34=15$

$\Rightarrow \cos\theta=\frac{15}{30}=\frac{1}{2}$

$\therefore \theta=\cos^{-1}\frac{1}{2}$

$\theta=\cos^{-1}(\cos 60^\circ)$

$\therefore \theta=60^\circ$

5. *We have given that*

$\vec{a}\perp(\vec{b}+\vec{c}),\vec{b}\perp(\vec{c}+\vec{a})$ *and* $\vec{c}\perp(\vec{a}+\vec{b})$

$\Rightarrow \vec{a}.(\vec{b}+\vec{c})=0, \vec{b}.(\vec{c}+\vec{a})=0$

and $\vec{c}.(\vec{a}+\vec{b})=0$

$\Rightarrow \vec{a}.\vec{b}+\vec{a}.\vec{c}=0, \vec{b}.\vec{c}+\vec{b}.\vec{a}=0$

and $\vec{c}.\vec{a}+\vec{c}.\vec{b}=0$

By adding all, we get

$2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0$

Now

$|\vec{a}+\vec{b}+\vec{c}|^2=(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})$

$=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})$

$=5^2+4^2+3^2+0\ (\because \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0)$

$=25+16+9$

$|\vec{a}+\vec{b}+\vec{c}|^2=50$

$|\vec{a}+\vec{b}+\vec{c}|=\sqrt{50}=5\sqrt{2}$

6.

Let O be the origin. Then

$\overrightarrow{OA}=\hat{i}+2\hat{j}+7\hat{k},$

$\overrightarrow{OB}=2\hat{i}+6\hat{j}+3\hat{k}$

$\overrightarrow{OC}=3\hat{i}+10\hat{j}-\hat{k}$

$\therefore \overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OC}=(2\hat{i}+6\hat{j}+3\hat{k})-(3\hat{i}+10\hat{j}-\hat{k})$

$=-\hat{i}-4\hat{j}+4\hat{k}$

$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=(3\hat{i}+10\hat{j}-\hat{k})-(2\hat{i}+6\hat{j}+3\hat{k})$

$=\hat{i}+4\hat{j}-4\hat{k}$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=(3\hat{i}+10\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+7\hat{k})$

$=2\hat{i}+8\hat{j}-8\hat{k}$

$\therefore |\overrightarrow{AB}|=\sqrt{1+16+16}=\sqrt{33}$

$|\overrightarrow{BC}|=\sqrt{1+16+16}=\sqrt{33}$

$|\overrightarrow{AC}|=\sqrt{4+64+64}=\sqrt{4\times 33}=2\sqrt{33}$

$\because |\overrightarrow{AC}|=|\overrightarrow{AB}|+|\overrightarrow{BC}|$

Hence, points A, B and C are Collinear.