Chapter: 3	Matrices आव्यूह

बहुविकल्पीय प्रश्न (MCQ)

1. यदि $\begin{bmatrix} 1 & 2 \\ x & 3 \end{bmatrix} = \begin{bmatrix} y & z \\ 5 & 3 \end{bmatrix}$ हो तो x, y तथा z मान होगा–

(a) x=5, y=1, z=2
(b) x=2, y=1, z=5
(c) x=5, y=2, z=1
(d) x=1, y=5, z=2

If $\begin{bmatrix} 1 & 2 \\ x & 3 \end{bmatrix} = \begin{bmatrix} y & z \\ 5 & 3 \end{bmatrix}$ **then the value of x, y and z will be-**

(a) x=5, y=1, z=2
(b) x=2, y=1, z=5
(c) x=5, y=2, z=1
(d) x=1, y=5, z=2

2. **A and B are two matrices then $(A+B)^2$ is equal to -**

(a) $A^2 + 2AB + B^2$
(b) $A^2 + AB + BA + B^2$
(c) $A^2 + 2AB + 2BA + B^2$
(d) Not possible.

यदि A और B दो आव्यूह हो तो $(A+B)^2$ का मान होगा–

(a) $A^2 + 2AB + B^2$
(b) $A^2 + AB + BA + B^2$
(c) $A^2 + 2AB + 2BA + B^2$
(d) संभव नहीं हैं।

3. **If A and B are two matrices then (AB)' is equal to -**

(a) A'B'
(b) B'A'
(c) (BA)'
(d) Not defined.

यदि A और B दो आव्यूह हो तो (AB)' का मान होगा–

(a) A'B'
(b) B'A'
(c) (BA)'
(d) परिभाषित नहीं ।

4. **If A=$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ then which of the following is A'-**

(a) $\begin{bmatrix} 2 & 2 & 3 \\ 5 & 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 6 \\ 2 & 5 \\ 1 & 4 \end{bmatrix}$

यदि A=$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ हो तो निम्नांकित में कौन A' के समान है–

(a) $\begin{bmatrix} 2 & 2 & 3 \\ 5 & 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 6 \\ 2 & 5 \\ 1 & 4 \end{bmatrix}$

5. **If A and B are two matrices then (A+B)' is equal to -**

यदि A एंव B दो आव्यूह हो तो (A+B)' के बराबर होगा–

(a) A'+B'
(b) B'+A'
(c) (AB)'
(d) Option (a) and (b) / विकल्प (a) और (b)

6. **A square matrix B is said to be a symmetric Matrix if -**

एक वर्ग आव्यूह B सममित आव्यूह होगा यदि –

(a) B=B'
(b) B=-B'
(c) B=-B
(d) None of these / कोई नहीं

7. **A square matrix B is said to be a skew-symmetric matrix if -**

एक वर्ग आव्यूह B विषम सममित आव्यूह होगा यदि–

(a) B=B'
(b) B=-B'
(c) B=-B
(d) None of these / कोई नहीं

8. **A square matrix B is said to be a singular matrix if-**

एक वर्ग आव्यूह **B** अव्युत्क्रमणीय आव्यूह होगा यदि–

(a) $|B| = 0$ (b) $|B| = 1$

(c) $|B| = 2$ (d) $|B| = 3$

9. **A square matrix B is said to be a non singular matrix if-**

एक वर्ग आव्यह **B** व्युत्क्रमणीय आव्यूह होगा यदि–

(a) $|B| = 0$ (b) $|B| \neq 0$

(c) $|B| = 1$ (d) $|B| = 2$

10. **If A be a matrix then AA^{-1} is equal to-**

यदि **A** एक आव्यूह हो तो AA^{-1} का मान होगा–

(a) I / unit matrix / **एकांक आव्यूह**

(b) $A^{-1}A$

(c) $1/2$

(d) Not defined / **परिभाषित नहीं।**

11. **If A and B are two matrices then $(AB)^{-1}$ is-**

यदि **A** और **B** दो आव्यूह हो तो $(AB)^{-1}$ होगा–

(a) $B^{-1}A^{-1}$

(b) $A^{-1}B^{-1}$

(c) $(BA)^{-1}$

(d) Undefined / अपरिभाषित

12. **If A be a square matrix then A^{-1} will exist if-**

यदि **A** एक वर्ग आव्यूह हो तो A^{-1} संभव होगा यदि–

(a) A is a singular / अव्युत्क्रमणीय

(b) A is non-singular / व्युत्क्रमणीय

(c) Row-matrix / पंक्ति आव्यूह

(d) Column matrix / स्तम्भ आव्यूह

13. **Which of the following is a non-singular matrix?**

निम्नलिखित में कौन व्युत्क्रमणीय आव्यूह हैं ?

(a) $\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 4 & -6 \\ 6 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -6 \\ -6 & 9 \end{bmatrix}$ **(d)** $\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix}$

14. **Which of the following is a singular matrix ?**

निम्नलिखित में कौन अव्युत्क्रमणीय आव्यूह हैं ?

(a) $\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -6 \\ 6 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

15. **For which of the following the inverse matrix can be obtained ?**

निम्नलिखित में किसका व्युत्क्रम ज्ञात किया जा सकता हैं?

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

16. **A matrix A= $[a_{ij}]_{n \times n}$ is said to be symmetric if -**

एक आव्यूह **A=** $[a_{ij}]_{n \times n}$ सममित आव्यूह होगा यदि–

(a) $a_{ij} = 0$

(b) $a_{ij} = -a_{ji}$

(c) $a_{ij} = a_{ji}$

(d) $a_{ij} = 1$

17. **A matrix A=$[a_{ij}]_{n\times n}$ is said to be a skew-symmetric if -**

एक आव्यूह **A=**$[a_{ij}]_{n\times n}$ विषम सममित आव्यूह होगा यदि –

(a) $a_{ij} = 0$

(b) $a_{ij} = -a_{ji}$ and $a_{ii} = 0$

(c) $a_{ij} = a_{ji}$

(d) $a_{ij} = 1$

MATRICES (2 MARKS QUESTIION)

1. **For which values of x and y the given metrices are equal-**

$$\begin{bmatrix} 0 & x-2 \\ 8 & 4 \end{bmatrix} \text{ and } \begin{bmatrix} 3y+7 & 5 \\ x+1 & 2-3y \end{bmatrix}$$

x और y के कौन से मान के लिए दिया गया आव्यूह समान होंगे–

$$\begin{bmatrix} 0 & x-2 \\ 8 & 4 \end{bmatrix} \text{ और } \begin{bmatrix} 3y+7 & 5 \\ x+1 & 2-3y \end{bmatrix}$$

2.Find the value of x from the following -

निम्न से x का मान ज्ञात कीजिए –

$$\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$$

3. If A = $\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix}$ and B= $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ then find the matrix (2A - 3B)

यदि A= $\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix}$ और B= $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ तो (2A - 3B) को ज्ञात करे –

4. Evaluate / मान ज्ञात करें–

$$3\begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 6 \end{bmatrix} - 2\begin{bmatrix} 2 & 5 & 7 \\ 3 & 4 & 8 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$$

5. Construct a 2x2 matrix whoes elements are given by $a_{ij} = (2_i - j)$

एक 2x2 आव्यूह की रचना करें जहाँ $a_{ij} = (2_i - j)$ हैं।

6. Construct a 3x3 matrix where $a_{ij} = {}^{i}\!/_{j}$

एक 3x3 आव्यूह की रचना करें जहाँ $a_{ij} = {}^{i}\!/_{j}$ हो.

7. Give an example of 3x3 matrix which is a zero matrix ?

एक 3x3 शून्य आव्यूह का उदाहरण दें ?

8. If A= $\begin{bmatrix} 2 & 3 & 5 \\ -1 & 0 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} 4 & -2 & 3 \\ 2 & 6 & -1 \end{bmatrix}$ then verify that A+B=B+A ?

यदि A= $\begin{bmatrix} 2 & 3 & 5 \\ -1 & 0 & 4 \end{bmatrix}$ और B= $\begin{bmatrix} 4 & -2 & 3 \\ 2 & 6 & -1 \end{bmatrix}$ तो सिद्ध करें कि A+B=B+A ?

9. Find a matrix X such that 2A+B+X=0, where A= $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and B= $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$?

आव्यूह X का मान ज्ञात करें यदि 2A+B+X=0 जहाँ A= $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ और B= $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$?

MATRICES (3 Marks Question)

1. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 30 & 3 & i \end{vmatrix} = x + iy$ then find the value of x and y?

यदि $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 30 & 3 & i \end{vmatrix} = x + iy$ हो तो x और y का मान ज्ञात करें ?

2. By using elementary row operation find the inverse of the matrix-

प्रारंभिक पंक्ति संक्रियाओं का प्रयोग करते हुए आव्यूह का व्युत्क्रम ज्ञात करें–

$$A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

3. If A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}$ then (AB)=

यदि A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ और B = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}$ तो (AB)=

4. If A= $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ then show that $A^3 = 4A$?

यदि A= $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ हो तो दिखाएँ कि $A^3 = 4A$?

5. Find the value of / मान निकालें–

$$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} =$$

MATRICES (5 Markes question)

1. If A= $\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ then find the value of k such that $A^2 = 8A + kI$

यदि A= $\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ तो k का वह मान निकाले जिसके लिए $A^2 = 8A + kI$ हो।

2. If A = $\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ then show that $A^2 - 5A - 14I = 0$?

यदि A = $\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ हो तो दर्शाएँ कि $A^2 - 5A - 14I = 0$?

3. If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then prove that -

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix};\ n \in N .$$

यदि $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ तो सिद्ध करे कि

$$A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix};\ n\in N.$$

4. If $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}=0$ then find the value of x?

यदि $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}=0$, हो तो x का मान ज्ञात करें ?

5. If $A=\begin{bmatrix}-3\\ 5\\ 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 6 & -4\end{bmatrix}$ then verify that (AB)'=B'A' ?

यदि $A=\begin{bmatrix}-3\\ 5\\ 2\end{bmatrix}$ और $B=\begin{bmatrix}1 & 6 & -4\end{bmatrix}$ हो तो दर्शाएँ कि (AB)'=B'A' ?

6. If matrix $A=\begin{bmatrix}1 & 3 & 5\\ -6 & 8 & 3\\ -4 & 6 & 5\end{bmatrix}$ then find the corrresponding symmetric part and skew-symmetric part of the matrix?

यदि $A=\begin{bmatrix}1 & 3 & 5\\ -6 & 8 & 3\\ -4 & 6 & 5\end{bmatrix}$ हो तो A के संगत सममित आव्यूह तथा विसम सममित आव्यूह को लिखें?

7. By using elementary row operation find the inverse of the matrix

$$A=\begin{bmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{bmatrix}$$

प्रारंभिक पंक्ति संक्रियाओं का प्रयोग करते हुए आव्यूह का व्युत्क्रम ज्ञात करें –

$$A=\begin{bmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{bmatrix}$$

8. If $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$ then prove that $|adj\ A|=|A|^2$

यदि $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$ तो सिद्ध करें कि $|adj\ A|=|A|^2$

9. If $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$ then find the matrices (A + A') and (A - A'). Also prove that (A+A') is a symmetric matrix while (A - A') is a skew-symmetric matrix ?

यदि $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$ तो आव्यूह (A+A') और (A-A') ज्ञात करें ।

पुन: सिद्ध करें कि (A+A') सममित आव्यूह है किन्तु (A-A') एक विषम सममित आव्यूह है ।

10. If $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ then prove that f(x+y)=f(x).f(y).

यदि $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ तो सिद्ध करें कि f(x+y)=f(x).f(y).

MATRICES (SOLUTION)

MCQ:-

1) (a)
2) (b)
3) (b)
4) (c)
5) (d)
6) (a)
7) (b)
8) (a)
9) (b)
10) (a)
11) (a)
12) (b)
13) (b)
14) (a)
15) (c)
16) (c)
17) (b)

2 Marks Solution

1) $\begin{bmatrix} 0 & x-2 \\ 8 & 4 \end{bmatrix} = \begin{bmatrix} 3y+7 & 5 \\ x+1 & 2-3y \end{bmatrix}$

$\because$ two matrices are equal if corresponding position elements are same.

$\therefore$ 3y+7=0 -------(1)
x-2=5 -------(2)
x+1=8 -------(3)
and 2-3y=4 ---(4)

eq^n-(1) $\Rightarrow y = \frac{-7}{3}$

eq^n-(2) $\Rightarrow$ x=7

eq^n-(3) $\Rightarrow$ x=7

eq^n-(4) $\Rightarrow y = \frac{-2}{3}$

$\because y = \frac{-7}{3} \neq \frac{-2}{3}$

hence for no value of x and y given matrices are equal

(2). $\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$

$\because$ 2x-y = 6 ------(1)
and y= -2 -----(2)

$\therefore$ (1)$\Rightarrow$ x = 2 and y = - 2

(3) $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

$\therefore$ (2A-3B) = $2\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 6 & 2 \\ 0 & -2 & 10 \end{bmatrix} - \begin{bmatrix} 3 & 6 & -3 \\ 0 & -3 & 9 \end{bmatrix}$

$= \begin{bmatrix} 4-3 & 6-6 & 2+3 \\ 0-0 & -2+3 & 10-9 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \end{bmatrix}$

(4) $3\begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 6 \end{bmatrix} - 2\begin{bmatrix} 2 & 5 & 7 \\ 3 & 4 & 8 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 3 & 9 & 12 \\ 6 & 15 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 10 & 14 \\ 6 & 8 & 16 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 8 \\ 8 & 4 & -12 \end{bmatrix}$

$= \begin{bmatrix} 3-4+4 & 9-10+0 & 12-14+8 \\ 6-6+8 & 15-8+4 & 18-16-12 \end{bmatrix}$

$= \begin{bmatrix} 3 & -1 & 6 \\ 8 & 11 & -10 \end{bmatrix}$

(5)

$\because a_{ij} = (2i - j)$

$\therefore \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2} = \begin{bmatrix} 2\times1-1 & 2\times1-2 \\ 2\times2-1 & 2\times2-2 \end{bmatrix}_{2\times2}$

$= \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}_{2\times2}$

(6) $\because a_{ij} = {}^{i}/_{j}$

$\therefore \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1/1 & 1/2 & 1/3 \\ 2/1 & 2/2 & 2/3 \\ 3/1 & 3/2 & 3/1 \end{bmatrix}_{3X3}$

$= \begin{bmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 2/3 \\ 3 & 3/2 & 1 \end{bmatrix}_{3X3}$

(7)

$0_{3X3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3X3}$

(8)

$A = \begin{bmatrix} 2 & 3 & 5 \\ -1 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 & 3 \\ 2 & 6 & -1 \end{bmatrix}$

$\therefore A+B = \begin{bmatrix} 2 & 3 & 5 \\ -1 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 4 & -2 & 3 \\ 2 & 6 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2+4 & 3-2 & 5+3 \\ -1+2 & 0+6 & 4-1 \end{bmatrix}$

$= \begin{bmatrix} 6 & 1 & 8 \\ 1 & 6 & 3 \end{bmatrix}$ ---------- (1)

and $B+A = \begin{bmatrix} 4 & -2 & 3 \\ 2 & 6 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 5 \\ -1 & 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} 4+2 & -2+3 & 3+5 \\ 2-1 & 6+0 & -1+4 \end{bmatrix}$

$= \begin{bmatrix} 6 & 1 & 8 \\ 1 & 6 & 3 \end{bmatrix}$ ---------- (2)

from eqn (1) and (2)
A+B=B+A

(9)

$$A=\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix} \text{ and } B=\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}$$

$$\because 2A+B+X=0$$

$$\therefore X=-(2A+B)$$

$$=-\left\{2\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}\right\}$$

$$=-\left\{\begin{bmatrix}-2 & 4\\ 6 & 8\end{bmatrix}+\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}\right\}$$

$$=-\begin{bmatrix}-2+3 & 4-2\\ 6+1 & 8+5\end{bmatrix}$$

$$=-\begin{bmatrix}1 & 2\\ 7 & 13\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -2\\ -7 & -13\end{bmatrix}$$

3 Marks solution :-

(1)

$$\begin{vmatrix}6i & -3i & 1\\ 4 & 3i & -1\\ 30 & 3 & i\end{vmatrix}=x+iy$$

$$\Rightarrow 6i(-3+3)+3i(4i+30)+(12-90i)=x+iy$$

$$\Rightarrow 0-12+90i+12-90i=x+iy$$

$$\Rightarrow 0=x+iy$$

$$\Rightarrow x=0 \text{ and } y=0$$

(2)

$$A=\begin{bmatrix}3 & -1\\ -4 & 2\end{bmatrix}$$

as, $A = IA$

$$\Rightarrow \begin{bmatrix}3 & -1\\ -4 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.A$$

$$R1 \to \frac{R1}{3}$$

$$\Rightarrow \begin{bmatrix}1 & -1/3\\ -4 & 2\end{bmatrix}=\begin{bmatrix}1/3 & 0\\ 0 & 1\end{bmatrix}.A$$

$$R2 \to R2+4R1$$

$$\Rightarrow \begin{bmatrix}1 & -1/3\\ 0 & 2-4/3\end{bmatrix}=\begin{bmatrix}1/3 & 0\\ 4/3 & 1\end{bmatrix}.A$$

$$\begin{bmatrix}1 & -1/3\\ 0 & 2/3\end{bmatrix}=\begin{bmatrix}1/3 & 0\\ 4/3 & 1\end{bmatrix}.A$$

$$R2 \to \frac{3}{2}R2$$

$$\Rightarrow \begin{bmatrix}1 & -1/3\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1/3 & 0\\ 2 & 3/2\end{bmatrix}.A$$

$$R1 \to R1+\frac{1}{3}R2$$

$$\Rightarrow \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{3}+\frac{2}{3} & \frac{1}{2}\\ 2 & \frac{3}{2}\end{bmatrix}.A$$

i.e $I_2 = A^{-1}.A$

$$\Rightarrow A^{-1}=\begin{bmatrix}1 & 1/2\\ 2 & 3/2\end{bmatrix}_{2X2}$$

(3)

$$\because A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix} \text{ and } B=\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\\ 1 & 3 & 2\end{bmatrix}$$

$$\therefore AB=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}.\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\\ 1 & 3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1+0+0 & 2+0+0 & 3+0+0\\ 0+6+0 & 0+4+0 & 0+2+0\\ 0+0+3 & 0+0+9 & 0+0+6\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2 & 3\\ 6 & 4 & 2\\ 3 & 9 & 6\end{bmatrix}$$

(4)

$$\because A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}.\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1+1 & -1-1\\ -1-1 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$$

$$A^3=A^2.A$$

$$=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}.\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2+2 & -2-2\\ -2-2 & 2+2\end{bmatrix}$$

$$=\begin{bmatrix}4 & -4\\ -4 & 4\end{bmatrix}$$

$$=4\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$\Rightarrow A^3=4A$$

(5)

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta & \sin\theta\cos\theta\\ -\sin\theta\cos & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I_2$$

5 Marks solution

(1)

$$\because A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$$

$$A^2=A.A$$

$$=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}.\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 0+0\\ -1-7 & 0+49\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 0\\ -8 & 49\end{bmatrix}$$

as, $A^2=8A+kI$

$$\Rightarrow \begin{bmatrix}1 & 0\\ -8 & 49\end{bmatrix}=8\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}+k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}8 & 0\\ -8 & 56\end{bmatrix}+k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\ -8 & 49\end{bmatrix}-\begin{bmatrix}8 & 0\\ -8 & 56\end{bmatrix}=k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1-8 & 0-0\\ -8+8 & 49-56\end{bmatrix}=k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix}=k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$(-7)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

hence $,k=-7$

(2)

$$A=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$$

$$\therefore A^2=A.A=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}.\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}9+20 & -15-10\\ -12-8 & 20+4\end{bmatrix}$$

$$A^2=\begin{bmatrix}29 & -25\\ -20 & 24\end{bmatrix}$$

Now, L.H.S $=A^2-5A-14I$

$$=\begin{bmatrix}29 & -25\\ -20 & 24\end{bmatrix}-5\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}-14\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}29 & -25\\ -20 & 24\end{bmatrix}-\begin{bmatrix}15 & -25\\ -20 & 10\end{bmatrix}-\begin{bmatrix}14 & 0\\ 0 & 14\end{bmatrix}$$

$$=\begin{bmatrix}29-15-14 & -25+25-0\\ -20+20-0 & 24-10-14\end{bmatrix}$$

$$\Rightarrow \text{L.H.S}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=\text{R.H.S Ans..}$$

(3)

$$\because A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$\Rightarrow A^2=A.A$$

$$=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}.\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta-\sin^2\theta & \sin\theta\cos\theta+\sin\theta\cos\theta\\ -\sin\theta\cos\theta-\sin\theta\cos\theta & -\sin^2\theta+\cos^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos2\theta & 2\sin\theta\cos\theta\\ -2\sin\theta\cos\theta & \cos2\theta\end{bmatrix}$$

$$A^2=\begin{bmatrix}\cos2\theta & \sin2\theta\\ -\sin2\theta & \cos2\theta\end{bmatrix}$$

$\therefore$ Again ,

$$A^3=A^2.A$$

$$=\begin{bmatrix}\cos2\theta & \sin2\theta\\ -\sin2\theta & \cos2\theta\end{bmatrix}.\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos\theta\cos2\theta-\sin\theta\sin2\theta & \sin\theta\cos2\theta+\cos\theta\sin2\theta\\ -\cos\theta\sin2\theta-\sin\theta\cos2\theta & -\sin\theta\sin2\theta+\cos\theta\cos2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos(\theta+2\theta) & \sin(\theta+2\theta)\\ -\sin(\theta+2\theta) & \cos(\theta+2\theta)\end{bmatrix}$$

$$\Rightarrow A^3=\begin{bmatrix}\cos3\theta & \sin3\theta\\ -\sin3\theta & \cos3\theta\end{bmatrix}$$

similarly ,

$$A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix};\ n\epsilon N$$

(4)

$$\begin{bmatrix}1 & x & 1\end{bmatrix}_{1\times3}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & 2 & 5\end{bmatrix}_{3\times3}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}_{3\times1}=0$$

$$\Rightarrow\begin{bmatrix}1+4x+3 & 2+5x+2 & 3+6x+5\end{bmatrix}_{1\times3}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}_{3\times1}=0$$

$$\Rightarrow[(1+4x+3)-2(2+5x+2)+3(3+6x+5)]=0$$

$$\Rightarrow(4x+4)-2(5x+4)+3(6x+8)=0$$

$$\Rightarrow 4x+4-10x-8+18x+24=0$$

$$\Rightarrow(4x-10x+18x)+(4-8+24)=0$$

$$\Rightarrow 12x+20=0$$

$$\Rightarrow\therefore x=-\frac{20}{12}=-\frac{5}{3}$$

(5)

$$\because A=\begin{bmatrix}-3\\5\\2\end{bmatrix} \text{ and } B=\begin{bmatrix}1 & 6 & -4\end{bmatrix}$$

$$\therefore A.B=\begin{bmatrix}-3\\5\\2\end{bmatrix}_{3\times1}.\begin{bmatrix}1 & 6 & -4\end{bmatrix}_{1\times3}$$

$$\Rightarrow A.B=\begin{bmatrix}-3 & -18 & 12\\5 & 30 & -20\\2 & 12 & -8\end{bmatrix}_{3\times3}$$

$$\therefore (AB)'=\begin{bmatrix}-3 & 5 & 2\\-18 & 30 & 12\\12 & -20 & -8\end{bmatrix} \text{---(i)}$$

Now,

$$B'A'=\begin{bmatrix}1\\6\\-4\end{bmatrix}_{3\times1}\begin{bmatrix}-3 & 5 & 2\end{bmatrix}_{1\times3}$$

$$=\begin{bmatrix}-3 & 5 & 2\\-18 & 30 & 12\\12 & -20 & -8\end{bmatrix}_{3\times3} \text{---(ii)}$$

from equqtion (i) and (ii)

$(AB)' = B'A'$

(6)

$$\because A=\begin{bmatrix}1 & 3 & 5\\-6 & 8 & 3\\-4 & 6 & 5\end{bmatrix}_{3\times3}$$

$$A'=\begin{bmatrix}1 & -6 & -4\\3 & 8 & 6\\5 & 3 & 5\end{bmatrix}_{3\times3}$$

Hence corresponding symmetric part

$$=\frac{1}{2}(A+A')$$

$$=\frac{1}{2}\left\{\begin{bmatrix}1 & 3 & 5\\-6 & 8 & 3\\-4 & 6 & 5\end{bmatrix}+\begin{bmatrix}1 & -6 & -4\\3 & 8 & 6\\5 & 3 & 5\end{bmatrix}\right\}$$

$$=\frac{1}{2}\left\{\begin{bmatrix}1+1 & 3-6 & 5-4\\-6+3 & 8+8 & 3+6\\-4+5 & 6+3 & 5+4\end{bmatrix}\right\}$$

$$=\frac{1}{2}\begin{bmatrix}2 & -3 & 1\\-3 & 16 & 9\\1 & 9 & 9\end{bmatrix}$$

Agian

$$\text{Skew-symmetric part}=\frac{1}{2}(A-A')$$

$$=\frac{1}{2}\left\{\begin{bmatrix}1 & 3 & 5\\-6 & 8 & 3\\-4 & 6 & 5\end{bmatrix}-\begin{bmatrix}1 & -6 & -4\\3 & 8 & 6\\5 & 3 & 5\end{bmatrix}\right\}$$

$$=\frac{1}{2}\begin{bmatrix}\cancel{1}-\cancel{1} & 3+6 & 5+4\\-6-3 & \cancel{8}-\cancel{8} & 3-6\\-4-5 & 6-3 & \cancel{5}-\cancel{5}\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}0 & 9 & 9\\-9 & 0 & -3\\-9 & 3 & 0\end{bmatrix}$$

(7)

$$\because A=\begin{bmatrix}2 & -3 & 3\\2 & 2 & 3\\3 & -2 & 2\end{bmatrix}$$

as A = IA

$$\Rightarrow\begin{bmatrix}2 & -3 & 3\\2 & 2 & 3\\3 & -2 & 2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}.A$$

$R_1 \leftrightarrows R_3$

$$\Rightarrow\begin{bmatrix}3 & -2 & 2\\2 & 2 & 3\\2 & -3 & 3\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\0 & 1 & 0\\1 & 0 & 0\end{bmatrix}.A$$

$R_1 \to R_1 - R_3,\ R_2 \to R_2 - R_3$

$$\Rightarrow\begin{bmatrix}1 & 1 & -1\\0 & 5 & 0\\2 & -3 & 3\end{bmatrix}=\begin{bmatrix}-1 & 0 & 1\\-1 & 1 & 0\\1 & 0 & 0\end{bmatrix}.A$$

$R_3 \to R_3 - 2R_1$

$$\Rightarrow\begin{bmatrix}1 & 1 & -1\\0 & 5 & 0\\0 & -5 & 5\end{bmatrix}=\begin{bmatrix}-1 & 0 & 1\\-1 & 1 & 0\\3 & 0 & -2\end{bmatrix}.A$$

$R_2 \to \frac{R_2}{5}$

$$\Rightarrow\begin{bmatrix}1 & 1 & -1\\0 & 1 & 0\\0 & -5 & 5\end{bmatrix}=\begin{bmatrix}-1 & 0 & 1\\-1/5 & 1/5 & 0\\3 & 0 & -2\end{bmatrix}.A$$

$R_1 \to R_1 - R_2,\ R_3 \to R_3 + 5R_2$

$$\Rightarrow\begin{bmatrix}1 & 0 & -1\\0 & 1 & 0\\0 & 0 & 5\end{bmatrix}=\begin{bmatrix}-4/5 & -1/5 & 1\\-1/5 & 1/5 & 0\\2 & 1 & -2\end{bmatrix}.A$$

$R_3 \to \frac{R_3}{5}$

$$\Rightarrow\begin{bmatrix}1 & 0 & -1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}-4/5 & -1/5 & 1\\-1/5 & 1/5 & 0\\2/5 & 1/5 & -2/5\end{bmatrix}.A$$

$R_1 \to R_1 + R_3$

$$\Rightarrow\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}-2/5 & 0 & 3/5\\-1/5 & 1/5 & 0\\2/5 & 1/5 & -2/5\end{bmatrix}.A$$

$$Hence,\ A^{-1}=\begin{bmatrix}-2/5 & 0 & 3/5\\-1/5 & 1/5 & 0\\2/5 & 1/5 & -2/5\end{bmatrix}$$

(8)

$\because\ A=\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}$

$\therefore |A|=\begin{vmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{vmatrix}$

$=1(8-6)+(0+9)+2(0-6)$

$=2+9-12$

$|A|=-1$ ---- (i)

Corresponding, minors of matrix 'A' is -

$a_{11}=\begin{vmatrix}2&-3\\-2&4\end{vmatrix}=8-6=2$

$a_{12}=\begin{vmatrix}0&-3\\3&4\end{vmatrix}=0+9=9$

$a_{13}=\begin{vmatrix}0&2\\3&-2\end{vmatrix}=0-6=-6$

$a_{21}=\begin{vmatrix}-1&2\\-2&4\end{vmatrix}=-4+4=0$

$a_{22}=\begin{vmatrix}1&2\\3&4\end{vmatrix}=4-6=-2$

$a_{23}=\begin{vmatrix}1&-1\\3&-2\end{vmatrix}=-2+3=1$

$a_{31}=\begin{vmatrix}-1&2\\2&-3\end{vmatrix}=3-4=-1$

$a_{32}=\begin{vmatrix}1&2\\0&-3\end{vmatrix}=-3-0=-3$

$a_{33}=\begin{vmatrix}1&-1\\0&2\end{vmatrix}=2-0=2$

$\therefore$ cofactor matrix $=\begin{bmatrix}2&-9&-6\\0&-2&-1\\-1&3&2\end{bmatrix}$

$\therefore \text{adj}(A)=[\text{cofactor matrix}]'=\begin{bmatrix}2&0&-1\\-9&-2&3\\-6&-1&2\end{bmatrix}$

$\Rightarrow |\text{adj}(A)|=2(-4+3)-0-(9-12)$

$=2\times(-1)-(-3)$

$=-2+3=1$

$=(-1)^2$

$\Rightarrow |\text{adj}(A)|=|A|^2$ [by uisng eqn (i)]

(9)

$\because\ A=\begin{bmatrix}0&1&-1\\-1&0&1\\1&-1&0\end{bmatrix}\Rightarrow A'=\begin{bmatrix}0&-1&1\\1&0&-1\\-1&1&0\end{bmatrix}$

$\Rightarrow A+A'=\begin{bmatrix}0&1&-1\\-1&0&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}0&-1&1\\1&0&-1\\-1&1&0\end{bmatrix}$

$=\begin{bmatrix}0+0&1-1&-1+1\\-1+1&0+0&1-1\\1-1&-1+1&0+0\end{bmatrix}$

$\Rightarrow (A+A')=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$ --- (i)

and $A-A'=\begin{bmatrix}0&1&-1\\-1&0&1\\1&-1&0\end{bmatrix}-\begin{bmatrix}0&-1&1\\1&0&-1\\-1&1&0\end{bmatrix}$

$=\begin{bmatrix}0-0&1+1&-1-1\\-1-1&0-0&1+1\\1+1&-1-1&0-0\end{bmatrix}$

$\Rightarrow (A-A')=\begin{bmatrix}0&2&-2\\-2&0&2\\2&-2&0\end{bmatrix}$ ---- (ii)

Again,

$\because (A+A')'=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}'=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

$\Rightarrow (A+A')'=(A+A')$

$\Rightarrow (A+A')$ is a symmetric matrix

Also,

$(A-A')'=\begin{bmatrix}0&2&-2\\-2&0&2\\2&-2&0\end{bmatrix}'=\begin{bmatrix}0&-2&2\\2&0&-2\\-2&2&0\end{bmatrix}$

$=-\begin{bmatrix}0&2&-2\\-2&0&2\\2&-2&0\end{bmatrix}$

$\Rightarrow (A-A')'=-(A-A')$

$\Rightarrow (A-A')$ is a skew - symmetric matrix.

10).

$$\because F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x+y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix} \text{---- (i)}$$

$$f(x+y)=\begin{bmatrix}\cos x\cos y-\sin x\sin y & -\sin x\cos y-\cos x\sin y & 0\\ \sin x\cos y+\cos x\sin y & \cos x\cos y-\sin x\sin y & 0\\ 0 & 0 & 1\end{bmatrix}$$

Again,

$$f(x).f(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}.\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos x\cos y-\sin x\sin y & -\sin y\cos x-\sin x\cos y & 0\\ \sin x\cos y+\cos x\sin y & -\sin x\sin y+\cos x\cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x).f(y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix} \text{---- (ii)}$$

from eqn (i) and (ii) we get ,

$f(x+y)=f(x).f(y)$