Chapter-11	Three- Dimensional Geometry त्रि-विमीय ज्यामिती

MCQ:- (बहुविकल्पीय प्रश्न) -

Q 1 If a line makes angles $90^\circ, 60^\circ$ and 30° with the +ve direction of the axises x,y,and z respectively. then find the direction cosine .

यदि एक रेखा x,y और z अक्षो की धनात्मक दिशा के साथ क्रमशः $90^\circ, 60^\circ$ तथा 30° का कोण बनाती है तो दिक्-कोसाइन ज्ञात करें।

(a) $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$ (b) $1, \frac{\sqrt{3}}{2}, \frac{1}{2}$

(c) 0 , 0 , 1 (d) $1, \frac{1}{\sqrt{2}}, \frac{1}{2}$

Q 2 A line makes equal angles with the axes. then the direction cosines are-

एक रेखा की दिक्-कोसाइन क्या होगी? जो अक्षों के साथ समान कोण बनाती है।

(a) $\pm 1, \pm 1, \pm 1$

(b) $\pm \frac{1}{\sqrt{2}}, \pm \frac{1}{\sqrt{2}}, \pm \frac{1}{\sqrt{2}}$

(c) 0 , 0 , 0

(d) $\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}$

Q 3 The direction cosines of a line whose direction ratios are 2, -6, 3.

यदि एक रेखा के दिक्-अनुपात 2,-6,3 हो तो इसकी दिक्-कोसाइन क्या होगा ?

(a) $\frac{2}{\sqrt{5}}, \frac{-6}{\sqrt{5}}, \frac{3}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{3}}, \frac{-6}{\sqrt{3}}, \frac{3}{\sqrt{3}}$

(c) $\frac{2}{7}, \frac{-6}{7}, \frac{3}{7}$ (d) $\frac{2}{49}, \frac{-6}{49}, \frac{3}{49}$

Q 4 The direction cosines of the line segment joining the point A(-2,4,5) and B(1,2,3) are

बिन्दूओं A (-2,4,5) और B(1,2,3) को मिलाने वाली रेखा की दिक्-कोसाइन होगा।

(a) $\frac{-3}{77}, \frac{2}{77}, \frac{-8}{77}$ (b) $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

(c) $\frac{-2}{\sqrt{77}}, \frac{4}{\sqrt{77}}, \frac{-5}{\sqrt{77}}$ (d) $\frac{1}{77}, \frac{2}{77}, \frac{3}{77}$

Q 5 The direction ratio of the vector $2\hat{i} + \hat{j} - 2\hat{k}$ are

सदिश $2\hat{i} + \hat{j} - 2\hat{k}$ का दिक्-अनुपात होगा।

(a) 2 , 1 , -2 (b) $\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$

(c) $\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-2}{\sqrt{3}}$ (d) $\frac{-2}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-2}{\sqrt{3}}$

Q 6 The direction cosine of the vector $-\hat{i} - 2\hat{j} - 2\hat{k}$ is

सदिश $-\hat{i} - 2\hat{j} - 2\hat{k}$ का दिक्-कोसाइन होगा।

(a) $\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ (b) -1, - 2, - 2

(c) $\frac{-1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}$ (d) $\frac{-1}{3}, \frac{-2}{3}, \frac{-2}{3}$

Q 7 The direction cosines of the y-axis are

y-अक्ष का दिक्-कोसाइन होगा।

(a) $(0,0,0)$ (b) $(1,0,0)$

(c) $(0,1,0)$ (d) $(0,0,1)$

Q 8 The vector equation of the line passing through the points (-1,0,2) and (3,4,6) is :

बिन्दू(-1,0,2) और (3,4,6) से होकर जाने वाली रेखा का सदिश समीकरण होगा।

(a) $\hat{i} + 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

(b) $\hat{i} - 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

(c) $-\hat{i} + 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

(d) $-\hat{i} + 2\hat{k} + \lambda(4\hat{i} - 4\hat{j} - 4\hat{k})$

Q 9 The cartesian equation of a line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ then the vector form of the equation is -

कार्तीय समीकरण $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ का सदिश समीकरण क्या होगा ?

(a) $(-5\hat{i} + 4\hat{j} + 6\hat{k}) - \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

(b) $(-5\hat{i} + 4\hat{j} - 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

(c) $(5\hat{i} + 4\hat{j} + 6\hat{k}) - \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

(d) $(5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

Q 10 The vector equation

$\vec{r} = (-3\hat{i} + 5\hat{j} - 6\hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 2\hat{k})$ then the cartesian form of the equation is

सदिश समीकरण $\vec{r} = (-3\hat{i} + 5\hat{j} - 6\hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 2\hat{k})$ को कार्तीय समीकरण रूप होगा।

(a) $\frac{x-3}{2} = \frac{y+5}{4} = \frac{z-6}{2}$

(b) $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$

(c) $\frac{x+3}{-2} = \frac{y+5}{-4} = \frac{z+6}{-2}$

(d) None of these (इनमें से कोई नहीं)

Q 11 The distance of the plane 2x-3y+6z+7=0 from the point (2,-3,-1) is

समतल 2x-3y+6z+7=0 से बिन्दु (2,-3,-1) की दूरी होगी।

(a) 4 (b) 3

(c) 2 (d) $\frac{1}{5}$

Q 12 The direction cosines of the normal to the plane 2x-3y-6z-3=0 are-

समतल 2x-3y-6z-3=0 के अभिलंब की दिक्-कोसाइन होगा।

(a) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$

(b) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

(c) $\frac{-2}{7}, \frac{-3}{7}, \frac{-6}{7}$

(d) None of these (इनमें से कोई नहीं)

Q 13 If 2x+5y-6z+3=0 be the equation of plane then the equation of any plane parallel to the given plane is

यदि 2x+5y-6z+3=0 एक समतल का समीकरण हो तो इसके समान्तर समतल का एक समीकरण होगा।

(a) 3x+5y-6z+3=0

(b) 2x-5y-6z+3=0

(c) 2x+5y-6z+k=0

(d) None of these (इनमें से कोई नहीं)

Very Short Question (2 Marks)

Q 1 Show that the lines , whose direction cosines are $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$ & $\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$ are mutually Perpendicular.

दर्शाइए कि दिक्-कोसाइन $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$ और $\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$ वाली रेखाएँ परस्पर लम्बवत् है।

Q 2 Show that the line segment joining the point A(1,-1,2) and B(3,4,2) is perpendicular to the line segment joining the points C(0,3,2) and D(3,5,6).

दिखाइए कि बिन्दु A(1,-1,2) और B(3,4,2) को मिलाने वाली रेखाखण्ड , बिन्दु C(0,3,2) और D(3,5,6) को मिलाने वाली रेखाखण्ड के लम्बवत् है।.

Q 3 A line passes through the point (1,2,3) and is parallel to $3\hat{i} + 2\hat{j} - 2\hat{k}$ .Find the equation of the line in vector forms.

बिन्दु (1,2,3) से गुजरने वाली रेखा का सदिश समीकरण ज्ञात कीजिए जो सदिश $3\hat{i} + 2\hat{j} - 2\hat{k}$ के समान्तर है।

Q 4 Find the vector equation of the line passing through the points(3,-2,-5) and (3,-2,6).

बिन्दुओ (3,-2,-5) और (3,-2,6) से गुजरने वाली रेखा का सदिश समीकरण को ज्ञात कीजिए।

Q 5 Find the angle between the lines having direction ratios are 3,4,5 and 4,-3,5.

दिक् अनुपात 3,4,5 और 4,-3,5 के बीच का कोण ज्ञात करे।

Q 6 Find the distance of the plane 2x-3y+6z+14=0 from origin.

मूल बिन्दु से समतल 2x-3y+6z+14=0 की दूरी ज्ञात करें।

Q 7 Find the value of λ for which the lines

$$\frac{x-1}{1} = \frac{y-2}{\lambda} = \frac{z+1}{1} \text{ and } \frac{x+1}{-\lambda} = \frac{y+1}{2} = \frac{z-2}{2}$$

are perpendicular to each other.

λ का मान ज्ञात करे जिससे कि सरल रेखाएँ

$$\frac{x-1}{1} = \frac{y-2}{\lambda} = \frac{z+1}{1} \text{ और } \frac{x+1}{-\lambda} = \frac{y+1}{2} = \frac{z-2}{2}$$

परस्पर लम्ब हो।

Q 8 If α, β, γ be the angles, which a line makes with the positive direction of axes. Prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

यदि कोई रेखा धनात्मक नियामक अक्षो के साथ α, β, γ कोण बनाती हो, तो सिद्ध कीजिए कि $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Q 9 Find the direction cosines for a line that is perpendicular to each of the two lines whose direction ratios are (2,-1,2) and (3,0,1).

उस सरल रेखा के लिए दिक्-कोसाइन ज्ञात कीजिए। जो दिक् अनुपातों (2,-1,2) और (3,0,1) वाली दो सरल रेखाओ मे से प्रत्येक पर लम्ब है।

Short Question (3 Marks)

Q 1 Find the angle between the lines $\vec{r}=-2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and $\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$.

दिये गये रेखाओ $\vec{r}=-2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ और $\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$ के बीच का कोण ज्ञात करें।

Q 2 Find the angle between the lines

$$\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3} \text{ and } \frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$$

दिये गये रेखाओं

$$\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3} \text{ और } \frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$$

के बीच का कोण ज्ञात करें।

Q 3 Find the value of P, where lines

$$\frac{1-x}{3}=\frac{7y-14}{2P}=\frac{z-3}{2} \text{ and } \frac{7-7x}{3P}=\frac{y-5}{1}=\frac{6-z}{5}$$

are perpendicular.

P का मान ज्ञात कीजिए, जहाँ रेखाएँ

$$\frac{1-x}{3}=\frac{7y-14}{2P}=\frac{z-3}{2} \text{ और } \frac{7-7x}{3P}=\frac{y-5}{1}=\frac{6-z}{5}$$

परस्पर लम्ब हो।

Q 4 Find the distance between the lines $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$

रेखाओं $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ और $\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$ के बीच की दूरी ज्ञात कीजिए।

Q 5 Find the distance of a plane 2x-3y+4z-6=0 from the origin.

समतल 2x-3y+4z-6=0 की मूल बिन्दु से दूरी ज्ञात कीजिए।

Q 6 Find the angle between of the planes 2x+y-2z=5 and 3x-6y-2z=7 in vector method.

दो समतलो 2x+y-2z=5 और 3x-6y-2z=7 के बीच का कोण सदिश विधि द्वारा ज्ञात कीजिए।

Q 7 Find the angle between the planes x+y+2z=9 and 2x–y+z=6.

दो समतलो x+y+2z=9 और 2x–y+z=6 के बीच का कोण ज्ञात कीजिए।

Q 8 Find the value of λ for which the planes 2x-4y+3z=7 and x+2y+λz=18 are perpendicular to each other.

समतलों 2x-4y+3z=7 और x+2y+λz=18 एक दूसरे के लम्बवत् हो तो λ का मान ज्ञात करें।

Q 9 Find the equation of the plane passing through the point (1,4,-2) and parallel to the plane -2x+y-3z=0.

उस समतल का समीकरण ज्ञात कीजिए, जो बिन्दु (1,4,-2) से होकर जाती है एवं समतल -2x+y-3z=0 के समान्तर हो।

Q 10 Find the equation of the plane passing through the points (1,1,-1) , (6,4,-5) and (-4,-2,3).

बिन्दुओं (1,1,-1) , (6,4,-5) और (-4,-2,3) से गुजरने वाले समतल का समीकरण ज्ञात कीजिए।

Long Question (5 Marks)

Q 1 Find the shortest distance of lines $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$.

रेखाओं $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ और $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$ के बीच की दूरी ज्ञात कीजिए।

Q 2 Find the shortest distance of lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$.

रेखाओ $\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$ और $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$ के बीच की दूरी ज्ञात कीजिए।

Q 3 Find the shortest distance of lines $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and $\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$.

रेखाओ $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ और $\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$ के बीच की दूरी ज्ञात कीजिए।

Q 4 Find the shortest distance between the line

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1} \text{ and } \frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$$

रेखाओ $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ और $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ के बीच की न्यूनतम दूरी ज्ञात कीजिए।

Q 5 Find the equation of the plane passing through the intersection of the plane 3x-y+2z-4=0 and x+y+z-2=0 and the point (2,2,1).

उस समतल का समीकरण ज्ञात कीजिए, जो समतलो 3x-y+2z-4=0 और x+y+z-2=0 के प्रतिछेदन तथा बिन्दु (2,2,1) से होकर जाती है।

Q 6 Find the equation of the plane passing through the line of intersection of the planes x+y+z=1 and 2x+3y+4z=5 which is perpendicular to the plane x-y+z=0.

समतलो $x+y+z=1$ और $2x+3y+4z=5$ के प्रतिच्छेदन रेखा से होकर जाने वाली तथा समतल $x-y+z=0$ पर लम्बवत् समतल का समीकरण ज्ञात कीजिए।

Q 7 **Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=7$, $\vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ and the point (2,1,3).**

उस समतल का सदिश समीकरण ज्ञात कीजिए जो समतल $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=7$, $\vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ के प्रतिच्छेदन और बिन्दु (2,1,3) से होकर जाता है।

Q 8 **Find the angle between the planes, whose vector equation are $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=5$ and $\vec{r}.(3\hat{i}-3\hat{j}+5\hat{k})=3$.**

समतलो,जिनके सदिश समीकरण $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=5$ और $\vec{r}.(3\hat{i}-3\hat{j}+5\hat{k})=3$ के बीच का कोण ज्ञात करे।

Q 9 **Find the equation of the plane passing through the point (1,-2,4) , (3,-4,5) and perpendicular to the plane x+y-2z=6.**

उस तल का समीकरण ज्ञात कीजिए जो बिन्दुओं (1,−2,4) और (3, −4, 5) से गुजरती है तथा समतल $x+y-2z=6$ के लम्बवत् है।

MCQ 1-Marks Solution

Ans : -

1 - (a) 2 - (d) 3 - (c) 4 - (b) 5 - (a) 6 - (d) 7 - (c)

8 - (c) 9 - (d) 10 - (b) 11 - (c) 12 - (a) 13 - (c)

2-Marks Solution

1 Ans :

The direction cosines of first lines are

$\frac{12}{13}, -\frac{3}{13}, -\frac{4}{13}$

So that $l_1=\frac{12}{13}, m_1=-\frac{3}{13}, n_1=-\frac{4}{13}$

The direction cosines of second lines are

$\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$

So that $l_2=\frac{4}{13}, m_2=\frac{12}{13}, n_2=\frac{3}{13}$

Now ,

$$l_1l_2+m_1m_2+n_1n_2=\frac{12}{13}\cdot\frac{4}{13}+\frac{-3}{13}\cdot\frac{12}{13}+\frac{-4}{13}\cdot\frac{3}{13}$$
$$=\frac{48}{169}-\frac{36}{169}-\frac{12}{169}$$
$$=0$$

Hence, the both lines are mutually perpendicular.

2 Ans : -

The direction ratios of the lines AB are

(3-1),(4+1),(-2,-2) i.e. 2,5,-4

$\therefore\ a_1=2, b_1=5, c_1=-4$

Similarly the direction ratios of line CD are 3,2,4

$\therefore\ a_2=3, b_2=2, c_2=4$

Now $a_1a_2+b_1b_2+c_1c_2=2.3+5.2+(-4).4$
$$=6+10-16$$
$$=0$$

Hence the line AB and CD are mutually perpendicular.

3 Ans :<u>In vector form</u>

The given line the passes through the point A(1,2,3) and in parallel to the vector $\vec{m}=3\hat{i}+2\hat{j}-2\hat{k}$

the position vector of A is, $\vec{r_1}=\hat{i}+2\hat{j}+3\hat{k}$

Hence the required line is $\vec{r}=\vec{r_1}+\lambda\vec{m}$

$$=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k}) \text{ ....eqn(1)}$$

<u>In cartesian form</u>

Taking $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

Then equation (1) becomes

$$x\hat{i}+y\hat{j}+z\hat{k}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$$
$$\Rightarrow x\hat{i}+y\hat{j}+z\hat{k}$$
$$=(1+3\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3-2\lambda)\hat{k}$$

$$x=1+3\lambda \Rightarrow \lambda=\frac{x-1}{3}$$

$$y=2+2\lambda \Rightarrow \lambda=\frac{y-2}{2}$$

$$z=3-2\lambda \Rightarrow \lambda=\frac{z-3}{-2}$$

$$\Rightarrow \frac{x-1}{3}=\frac{y-2}{2}=\frac{z-3}{-2}=\lambda$$

Hence $\frac{x-1}{3}=\frac{y-2}{2}=\frac{z-3}{-2}$ are the required equation of the given line.

4 Ans : -

Let the position vectors of (3, -2, -5) and (3,-2,6) be $\vec{r_1}$ and $\vec{r_2}$ respectively . Then

$$\vec{r_1}=3\hat{i}-2\hat{j}-5\hat{k} \text{ and } \vec{r_2}=3\hat{i}-2\hat{j}+6\hat{k}$$

$\therefore \vec{r}_2 - \vec{r}_1 = (3\hat{i} - 2\hat{j} + 6\hat{k}) - (3\hat{i} - 2\hat{j} - 5\hat{k})$

$= 0\hat{i} + 0\hat{j} + 11\hat{k} = 11\hat{k}$

$\therefore$ The vector equation of the line is

$\vec{r} = \vec{r}_1 + \lambda(\vec{r}_2 - \vec{r}_1)$

i.e. $\vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k} - \lambda 11\hat{k}$

This is the required equation.

5 Ans : -

Let θ be the angle between the given vector

Let $a_1=3, b_1=4, c_1=5$

and $a_2=4, b_2=-3, c_2=5$

$$\therefore \cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\left(\sqrt{a_1^2 + b_1^2 + c_1^2}\right)\left(\sqrt{a_2^2 + b_2^2 + c_2^2}\right)}$$

$$= \frac{3.4 + 4.(-3) + 5.5}{\left(\sqrt{9+16+25}\right)\left(\sqrt{16+9+25}\right)}$$

$$\cos\theta = \frac{25}{\sqrt{50}\cdot\sqrt{50}} = \frac{25}{50} = \frac{1}{2}$$

$$\theta = \cos^{-1}\frac{1}{2} = \cos(\cos 60^\circ)$$

$$\therefore \quad \theta = 60^\circ$$

6 Ans : -

Let O be the origin.

i.e.$x_1=0=y_1=z_1$

the d.r.s of the given plane are a=2,b=-3,c=6

So that, perpendicular distance

$$P = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$= \left|\frac{2.0 + (-3).0 + 6.0 + 14}{\sqrt{2^2 + (-3)^2 + 6^2}}\right|$$

$$= \frac{14}{\sqrt{4+9+36}}$$

$$= \frac{14}{\sqrt{49}}$$

$P = \frac{14}{7} = 2$ units.

Hence, the distance of the plane 2x–3y+6z+14=0 from origin is 2 units.

7 Ans : -

The d.r.s of the first line are $a_1=1, b_1=\lambda, c_1=1$

The d.r.s of the second line are $a_2=-\lambda, b_2=2, c_2=2$

$\because$ the lines are perpendicular to each other.

so that, $\quad a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow 1.(-\lambda) + \lambda.2 + 1.2 = 0$

$\Rightarrow \quad -\lambda + 2\lambda + 2 = 0$

$\lambda = -2$

8 Ans : -

The given line makes angles α, β, γ with the x-axis, y-axis and z-axis respectively.

So that, the direction cosine are

$l = \cos\alpha$

$m = \cos\beta$

$n = \cos\gamma$

we know that , $\quad l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow 3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

9 Ans : -

Let l,m,n be the direction cosines of the required line.

Then , $l(2) + m(-1) + n(2) = 0$

and , $\quad l(3) + m(0) + n(1) = 0$

$\Rightarrow \quad 2l - m + 2n = 0 \quad$ ....(1)

and $\quad 3l - 0m + n = 0 \quad$ ....(2)

Solve the (1) and(2) by Cross multiplying

method,we get-

$$\frac{l}{-1} = \frac{m}{6-2} = \frac{n}{3}$$

$$\Rightarrow \frac{l}{-1} = \frac{m}{4} = \frac{n}{3} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{(-1)^2+4^2+3^2}} = \frac{1}{\sqrt{26}}$$

$$\therefore l = \frac{-1}{\sqrt{26}}, \; m = \frac{4}{\sqrt{26}}, \; n = \frac{3}{\sqrt{26}}$$

3-Marks Solution

1 Ans : - Let θ be the angle between the given lines .

The given lines are parallel to the vector.

$\overrightarrow{m_1} = 3\hat{i}+2\hat{j}+6\hat{k}$ and $\overrightarrow{m_2} = \hat{i}+2\hat{j}+2\hat{k}$

$$\therefore \cos\theta = \frac{\overrightarrow{m_1}.\overrightarrow{m_2}}{|\overrightarrow{m_1}||\overrightarrow{m_2}|} = \frac{(3\hat{i}+2\hat{j}+6\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{|3\hat{i}+2\hat{j}+6\hat{k}||\hat{i}+2\hat{j}+2\hat{k}|}$$

$$= \frac{3+4+12}{\sqrt{3^2+2^2+6^2}\sqrt{1^2+2^2+2^2}}$$

$$= \frac{19}{\sqrt{9+4+36}\sqrt{1+4+4}}$$

$$= \frac{19}{\sqrt{49}\sqrt{9}} = \frac{19}{7\cdot 3} = \frac{19}{21}$$

$$\cos\theta = \frac{19}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

2 Ans : -

The d.r.s. of the line $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ are 2,5,3

i.e. $a_1 = 2, b_1 = 5, c_1 = 3$

And , the d.r.s of the line $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ are -1,8,4

i.e. $a_2 = -1, b_2 = 8, c_2 = 4$

Let θ be the angle between the given lines

$$\therefore \cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{2.(-1)+5.8+3.4}{\sqrt{4+25+9}\,.\sqrt{1+64+16}}$$

$$\cos\theta = \frac{-2+40-12}{\sqrt{38}.9} = \frac{26}{9\sqrt{38}}$$

$$\therefore\ \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

3 Ans : -

The d.r.s of the line $\frac{1-x}{3} = \frac{7y-14}{2P} = \frac{z-3}{2}$

i.e. $\frac{x-1}{-3} = \frac{y-14}{\left(2P/7\right)} = \frac{z-3}{2}$ are -3, $\frac{2P}{7}$,2

The d.r.s of the line $\frac{7-7x}{3P} = \frac{y-5}{1} = \frac{6-z}{5}$

i.e. $\frac{x-1}{\left(-3P/7\right)} = \frac{y-5}{1} = \frac{z-6}{-5}$ are $-\frac{3P}{7}, 1, -5$

$\because$ the lines are perpendicular

So that $(-3)\cdot\left(-\frac{3P}{7}\right)+\frac{2P}{7}\cdot 1+2\cdot(-5) = 0$

$$\Rightarrow \frac{9P}{7}+\frac{2P}{7} = 10$$

$$\Rightarrow 11P = 70$$

$$\Rightarrow P = \frac{70}{11}$$

4 Ans : -

Comparing the given equation with the standard form of equation $\vec{r} = \vec{r_1}+\lambda\vec{m}$ and $\vec{r} = \vec{r_2}+\mu\vec{m}$ we, get

$\vec{r_1} = \hat{i}+2\hat{j}-4\hat{k}$

$\vec{r_2} = 3\hat{i}+3\hat{j}-5\hat{k}$

$\vec{m} = 2\hat{i}+3\hat{j}+6\hat{k}$

$\therefore |\vec{m}| = \sqrt{4+9+36} = \sqrt{49} = 7$

$\vec{r_2}-\vec{r_1} = 2\hat{i}+\hat{j}-\hat{k}$

$$\therefore \vec{m}\times(\vec{r_2}-\vec{r_1}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= (-3-6)\hat{i}+(12+2)\hat{j}+(2-6)\hat{k}$$
$$= -9\hat{i}+14\hat{j}-4\hat{k}$$

$$(\textit{shortest distance between the lines})\,d = \left|\frac{\vec{m}\times(\vec{r_2}-\vec{r_1})}{|\vec{m}|}\right|$$

$$= \frac{|-9\hat{i}+14\hat{j}-4\hat{k}|}{7}$$

$$= \frac{\sqrt{81+196+16}}{7}$$

$$= \frac{\sqrt{293}}{7}\ \text{units.}$$

5 Ans : -

Let O(0,0,0) be the origin. The d.r.s of given lines are a=2,b=-3,c=4

$\therefore$ The length of the perpendicular is

$$P = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$= \left|\frac{2.0+(-3).0+4.0-6}{\sqrt{4+9+16}}\right|$$

$$= \frac{|-6|}{\sqrt{29}}$$

$$= \frac{6}{\sqrt{29}}\ \text{units}$$

6 Ans : -

The given planes are

$3x - 6y - 2z = 7$ and $2x + y - 2z = 5$

So that, the normal of the planes are

$\vec{n_1} = 3\hat{i} - 6\hat{j} - 2\hat{k}$ and $\vec{n_2} = 2\hat{i} + \hat{j} - 2\hat{k}$

Since, angle between the planes is equal to angle between the normals

$$\therefore \cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$$

$$= \frac{6 - 6 + 4}{\sqrt{9+36+4}\sqrt{4+1+4}}$$

$$= \frac{4}{\sqrt{49}\sqrt{9}} = \frac{4}{7 \cdot 3}$$

$$\cos\theta = \frac{4}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

7 Ans : -

Let the θ be the angle between planes.

The eqn. of 1st plane is x+y+2z=9

so that, d.r.s are $a_1 = 1, b_1 = 1, c_1 = 2$

The eqn. of 2nd plane is 2x-y+z=6

so that, d.r.s are $a_2 = 2, b_2 = -1, c_2 = 1$

$$\therefore \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{2 - 1 + 2}{\sqrt{1+1+4}\sqrt{4+1+1}}$$

$$\cos\theta = \frac{3}{6} = \frac{1}{2} = \cos 60^\circ$$

$$\therefore \theta = 60^\circ$$

8 Ans : -

The given planes are

2x-4y+3z=7 and x+2y+λz=18

where $a_1 = 2, b_1 = -4, c_1 = 3$

and $a_2 = 1, b_2 = 2, c_2 = \lambda$

The given planes are perpendicular,

$$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$2 - 8 + 3\lambda = 0$$

$$\lambda = \frac{6}{3} = 2$$

9 Ans : -

Let the eqn. of a plane parallel to the planes

-2x+y-3z=0 is -2x+y-3z=k.

$\because$ planes -2x+y-3z=k is passes through

points (1,4,-2)

$\therefore \quad -2.1 + 4 - 3.(-2) = k$

$\Rightarrow \quad -2 + 4 + 6 = k$

$k = 8$

Hence the required plane is

$-2x + y - 3z = 8$

10 Ans : -

Let the equation of plane be

ax+by+cz+d=0 ....(1)

The plane is passes through the point(1,1,-1)

(6,4,-5) and (-4,-2,3) we, get

a+b-c+d=0 ....(2)

6a+4b-5c+d=0 ....(3)

and -4a-2b+3c+d=0 ....(4)

subtract (3)-(2) and (3)-(4) we, get

5a-5b-4c=0 ....(5)

and , 10a+6b-8c=0 ....(6)

solve eqn(5) and (6) by cross multiplication method we, get

$$\frac{a}{40+24} = \frac{b}{-40+40} = \frac{c}{30+50} = k(\text{say})$$

a=64k , b=0 , c=80k

put these values in eqn(2) we, get

64k+0-80k+d=0

d=16k

Again, put the values of a,b,c and d in eqn(1) we get

64kx+0y+80kz+16k=0

$\Rightarrow \quad 64x + 80z + 16 = 0$

$\Rightarrow \quad 4x + 5z + 1 = 0$

This is the required equation of plane.

5-Marks Solution

1 Ans : -

The equation of the given lines are

$$\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$$

and , $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$

we know that the shortest distance between the lines $\vec{r}=\vec{a_1}+\lambda b_1$ and $\vec{r}=\vec{a_2}+\mu b_2$ is, given by

$$d=\left|\frac{(\vec{b_1}\times\vec{b_2})(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right| \text{.............(1)}$$

comparing the given equation , we get

$\vec{a_1}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b_1}=\hat{i}-\hat{j}+\hat{k}$

$\vec{a_2}=2\hat{i}-\hat{j}-\hat{k}$

$\vec{b_2}=2\hat{i}+\hat{j}+2\hat{k}$

$\therefore\ \vec{a_2}-\vec{a_1}=(2\hat{i}-\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+\hat{k})$

$=\hat{i}-3\hat{j}-2\hat{k}$

$$\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 2 & 1 & 2\end{vmatrix}$$

$=(-2-1)\hat{i}-(2-2)\hat{j}+(1+2)\hat{k}$

$=-3\hat{i}+3\hat{k}$

$|\vec{b_1}\times\vec{b_2}|=\sqrt{(-3)^2+3^2}=\sqrt{9+9}=3\sqrt{2}$

Now ,

$$d=\left|\frac{(-3\hat{i}+3\hat{k})\cdot(\hat{i}-3\hat{j}-2\hat{k})}{3\sqrt{2}}\right|$$

$$=\left|\frac{-3-6}{3\sqrt{2}}\right|=\frac{9}{3\sqrt{2}}=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2} \text{ units.}$$

2 Ans : -

The equation of the given lines are

$$\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$$

and , $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

we know that the shortest distance between the lines $\vec{r}=\vec{a_1}+\lambda b_1$ and $\vec{r}=\vec{a_2}+\mu b_2$ is, given by

$$d=\left|\frac{(\vec{b_1}\times\vec{b_2})(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

comparing the given equation , we get

$\vec{a_1}=\hat{i}+\hat{j}$

$\vec{b_1}=2\hat{i}-\hat{j}+\hat{k}$

$\vec{a_2}=2\hat{i}-\hat{j}-\hat{k}$

$\vec{b_2}=3\hat{i}-5\hat{j}+2\hat{k}$

$\therefore\ \vec{a_2}-\vec{a_1}=(\hat{i}-2\hat{j}-\hat{k})$

$$\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -5 & 2\end{vmatrix}$$

$=(-2+5)\hat{i}-(4-3)\hat{j}+(-10+3)\hat{k}$

$=3\hat{i}-\hat{j}-7\hat{k}$

$|\vec{b_1}\times\vec{b_2}|=\sqrt{9+1+49}=\sqrt{59}$

Now ,

$$d=\left|\frac{(3\hat{i}-\hat{j}-7\hat{k})\cdot(\hat{i}-2\hat{j}-\hat{k})}{\sqrt{59}}\right|$$

$$=\left|\frac{3+2+7}{\sqrt{59}}\right|=\frac{12}{\sqrt{59}}=\frac{12\sqrt{59}}{59} \text{ units.}$$

3 Ans : -

The given lines are

$$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$$

and , $\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$

we know that the shortest distance between the lines $\vec{r}=\vec{a_1}+\lambda\vec{b}_1$ and $\vec{r}=\vec{a_2}+\mu\vec{b}_2$ is given by

$$d=\left|\frac{(\vec{b_1}\times\vec{b_2})(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

comparing the given equation , we get

$\vec{a_1}=\hat{i}+2\hat{j}+3\hat{k}$

$\vec{b_1}=\hat{i}-3\hat{j}+2\hat{k}$

$\vec{a_2}=4\hat{i}+5\hat{j}+6\hat{k}$

$\vec{b_2}=2\hat{i}+3\hat{j}+\hat{k}$

$\therefore\ \vec{a_2}-\vec{a_1}=(3\hat{i}+3\hat{j}+3\hat{k})$

$$\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 2\\ 2 & 3 & 1\end{vmatrix}$$

$=(-3-6)\hat{i}-(1-4)\hat{j}+(3+6)\hat{k}$

$=-9\hat{i}+3\hat{j}+9\hat{k}$

$|\vec{b_1}\times\vec{b_2}|=\sqrt{81+9+81}=\sqrt{171}=3\sqrt{19}$

Now ,

$$d=\left|\frac{\left(-9\hat{i}+3\hat{j}+9\hat{k}\right)\cdot\left(3\hat{i}+3\hat{j}+3\hat{k}\right)}{3\sqrt{19}}\right|$$

$$=\left|\frac{-27+9+27}{3\sqrt{19}}\right|=\frac{9}{3\sqrt{19}}=\frac{3}{\sqrt{19}} \text{ units.}$$

4 Ans : -

The given lines are $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

These lines are comparing with standard form of equation. we, get

$x_1=-1, y_1=-1, z_1=-1$

$x_2=3, y_2=5, z_2=7$

$a_1=7, b_1=-6, c_1=1$

$a_2=1, b_2=-2, c_2=1$

$$d=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$$

Now ,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$=\begin{vmatrix} 3+1 & 5+1 & 7+1 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}=\begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$=4(-6+2)-6(7-1)+8(-14+6)$

$=-16-36-64$

$=-116$

Again ,

$\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}$

$=\sqrt{(-6+2)^2+(1-7)^2+(-14+6)^2}$

$=\sqrt{16+36+64}=\sqrt{116}=2\sqrt{29}$

Subsitituting all the values in eqn(1). we, get

$$\therefore d=\frac{-116}{2\sqrt{29}}=\frac{-58}{\sqrt{29}}=\frac{-58\sqrt{29}}{29}=-2\sqrt{29}$$

$$=2\sqrt{29} \text{ unit } (\because \text{distance is always non - negative})$$

5 Ans : -

The equation of the plane passing through the intersection of the planes 3x-y+2z-4=0 and x+y+z-2=0 is

$(3x-y+2z-4)+\lambda(x+y+z-2)=0$, $\lambda\epsilon R$

......................(1)

The plane passes through the point (2,2,1), therefore this point will satisfy the (1).

$(3.2-2+2.1-4)+\lambda(2+2+1-2)=0$

$\Rightarrow(6-2+2-4)+\lambda(3)=0$

$\Rightarrow 2+3\lambda=0$

$\therefore \lambda=-2/3$

Substituting $\lambda=-2/3$ in eqn(1) , we obtain

$(3x-y+2z-4)-\frac{2}{3}(x+y+z-2)=0$

$\Rightarrow 9x-3y+6z-12-2x-2y-2z+4=0$

$\Rightarrow 7x-5y+4z-8=0$

This is the required equation of plane.

6 Ans : -

The equation of plane passing through the intersection of the planes x+y+z=1 and 2x+3y+4z=5 is

$(x+y+z-1)+\lambda(2x+3y+4z-5)=0, \lambda\epsilon R$

$=>(2\lambda+1)x+(3\lambda+1)y+(4\lambda+1)z+(-5\lambda-1)=0$

.......................(1)

The d.r.s a_1,b_1,c_1 of the plane (1) are

$(2\lambda+1),(3\lambda+1),(4\lambda+1)$ respectively.

The plane of(1) is perpendicular to x-y+z=0 Its d.r.s a_2,b_2,c_2 are 1,-1and 1 respectively.

$\because$ the planes are perpendicular

$\therefore$ $a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow(2\lambda+1)\cdot 1+(3\lambda+1)\cdot(-1)+(4\lambda+1)\cdot 1=0$

$\Rightarrow 2\lambda+1-3\lambda-1+4\lambda+1=0$

$\Rightarrow 3\lambda+1=0$

$\Rightarrow \lambda=\frac{-1}{3}$

put $\lambda=\frac{-1}{3}$ in(1) we get

$$\left(\frac{-2}{3}+1\right)x+\left(\frac{-3}{3}+1\right)y+\left(\frac{-4}{3}+1\right)z+\left(\frac{5}{3}-1\right)=0$$

$$\Rightarrow \frac{1}{3}x+0.y-\frac{1}{3}z+\frac{2}{3}=0$$

$$\Rightarrow x-z+2=0$$

This is required eqn. of plane.

7 Ans : -

The eqn. of planes are

$$\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)-7=0 \quad \text{.............(1)}$$

and $\vec{r}\cdot\left(2\hat{i}+5\hat{j}+3\hat{k}\right)-9=0$ ....(2)

The eqn. of plane passing through the intersection of the planes (1) and (2) is, given by

$$\left[\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)-7\right]+\lambda\left[\vec{r}\cdot\left(2\hat{i}+5\hat{j}+3\hat{k}\right)-9\right]=0$$

$$\Rightarrow \vec{r}\cdot\left[\left(2\hat{i}+2\hat{j}-3\hat{k}\right)+\lambda\left(2\hat{i}+5\hat{j}+3\hat{k}\right)\right]=9\lambda+7$$

$$\Rightarrow \vec{r}\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}\right]=9\lambda+7 \quad \text{....................(3)}$$

The plane passes through the point (2,1,3)

$\therefore$ its position vector is $\vec{r}=2\hat{i}+\hat{j}+3\hat{k}$

Subsitituting in eqn(3) we, obtain

$$\left(2\hat{i}+\hat{j}+3\hat{k}\right)\cdot\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}\right]=9\lambda+7$$

$$\Rightarrow 2(2+2\lambda)+(2+5\lambda)+3(-3+3\lambda)=9\lambda+7$$

$$\Rightarrow 4+4\lambda+2+5\lambda-9+9\lambda=9\lambda+7$$

$$\Rightarrow -3+18\lambda=9\lambda+7$$

$$\Rightarrow 9\lambda=10 \Rightarrow \lambda=\frac{10}{9}$$

Substituting $\lambda=\frac{10}{9}$ in eqn (3). we, get

$$\vec{r}\cdot\left[\left(2+\frac{20}{9}\right)\hat{i}+\left(2+\frac{50}{9}\right)\hat{j}+\left(-3+\frac{30}{9}\right)\hat{k}\right]=\frac{90}{9}+7$$

$$\Rightarrow \vec{r}\cdot\left[38\hat{i}+68\hat{j}+3\hat{k}\right]=153$$

This is required equation of plane.

8 Ans : -

Given equation of plane are

$$\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=5 \text{ and } \vec{r}\cdot\left(3\hat{i}-3\hat{j}+5\hat{k}\right)=3$$

We know that if $\vec{n_1}$ and $\vec{n_2}$ are normal to the planes $\vec{r}\cdot\vec{n_1}=d_1$ and $\vec{r}\cdot\vec{n_2}=d_2$ then the angle between them θ, is given by

$$\cos\theta=\left|\frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\cdot|\vec{n_2}|}\right| \text{........(1)}$$

Here, $\vec{n_1}=2\hat{i}+2\hat{j}-3\hat{k}$

$\vec{n_2}=3\hat{i}-3\hat{j}+5\hat{k}$

$\vec{n_1}\cdot\vec{n_2}=6-6-15=-15$

$|\vec{n_1}|=\sqrt{4+4+9}=\sqrt{17}$

$|\vec{n_2}|=\sqrt{9+9+25}=\sqrt{43}$

$$\therefore \cos\theta=\left|\frac{-15}{\sqrt{17}\cdot\sqrt{43}}\right|=\frac{15}{\sqrt{731}}$$

$$\therefore \theta=\cos^{-1}\frac{15}{\sqrt{731}} \quad \textit{Ans...}$$

9 Ans : -

Let the equation of required plane be

ax+by+cz+d=0 ....(1)

It is perpendicular to the plane x+y-2z=6

$\therefore \quad a\cdot 1+b\cdot 1+c\cdot(-2)=0$

$\Rightarrow \quad a+b-2c=0$ .... (2)

$\because$ the plane (1) passes through the given points(1,-2,4) and (3,-4,5). then,

a-2b+4c+d=0 ....(3)

and 3a-4b+5c+d=0 ....(4)

Subtract (3) from (4) we, gets

2a-2b+c=0 ....(5)

Now from (2) and (5)

a+b-2c=0

2a-2b+c=0

Solving these equation by cross multiplication method we, get

$$\frac{a}{1-4}=\frac{b}{-4-1}=\frac{c}{-2-2}$$

$$\Rightarrow \frac{a}{-3}=\frac{b}{-5}=\frac{c}{-4}=-k(\text{say})$$

$$\Rightarrow \frac{a}{3}=\frac{b}{5}=\frac{c}{4}=k$$

$\Rightarrow a=3k$, $b=5k$, $c=4k$

Substituting these values in (3). we, get
3k-10k+16k+d=0
9k+d=0
d=-9k
Putting the value of a,b,c and d in (1) we, get
3x+5y+4z–9 = 0

Hence, the required plane.