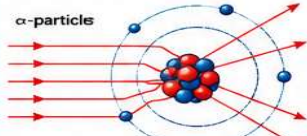


CHAPTER 12

ATOMS

1. RUTHERFORD'S ATOMIC MODEL

α -particle scattering experiment



On the basis of the α -particle scattering experiment, Rutherford made the following observations:

- The entire positive charge and almost entire mass of the atom is concentrated at its centre in a very tiny region of the order of 10^{-15} m, called nucleus.
- The negatively charged electrons revolve around the nucleus in different orbits.
- The total positive charge on nucleus is equal to the total negative charge on electrons. Therefore, atom as a whole is neutral.
- The centripetal force required by electron for revolution is provided by the electrostatic force of attraction between the electron and the nucleus.

2. SCATTERING GEOMETRY & RUTHERFORD FORMULA

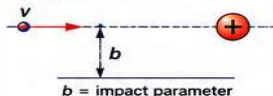
Distance of Closest Approach

$$r_0 = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{E_k}$$

where, E_k = kinetic energy of the α -particle.

Impact Parameter

The perpendicular distance of the velocity vector of α -particle from the central line of the nucleus, when the particle is far away from the nucleus is called impact parameter.

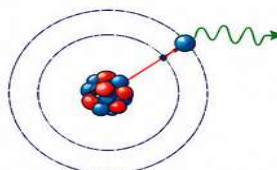


Rutherford's Scattering Formula

$$\frac{N(\theta)}{N_i} = \frac{nZ^2e^4t}{16\pi^2\epsilon_0^2E^2} \frac{1}{\sin^4(\theta/2)}$$

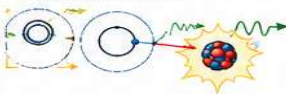
where, $N(\theta)$ = number of α -particles scattered at angle θ
 N_i = total number of α -particles reaching the screen
 n = number of atoms per unit volume in the foil
 Z = atomic number
 E = kinetic energy of the α -particles
 t = foil thickness

3. LIMITATIONS OF RUTHERFORD MODEL



1 About the Stability of Atom

According to Maxwell's electromagnetic wave theory, electron should emit energy in the form of electromagnetic wave during its orbital motion. Therefore, radius of orbit of electron will decrease gradually and ultimately it will fall in the nucleus.

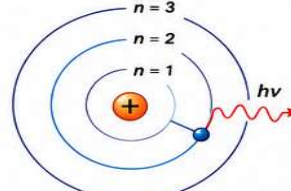


2 About the Line Spectrum

Rutherford atomic model cannot explain atomic line spectrum.



4. BOHR'S ATOMIC MODEL



• Electron can revolve in certain non-radiating orbits called stationary orbits for which the angular momentum of electron is an integer multiple of $\frac{h}{2\pi}$.

$$mvr = \frac{nh}{2\pi}$$

where $n = 1, 2, 3, \dots$ called principal quantum number.

• Radiation of energy occurs only when electron jumps from one permitted orbit to another permitted orbit.

$$hv = E_2 - E_1$$

where E_1 and E_2 are energies of electron in orbits.

5. QUANTITIES IN BOHR MODEL

	Radius of orbit	$r = \frac{n^2 h^2}{4\pi^2 m K Z e^2} \Rightarrow r \propto \frac{n^2}{Z}$	where, n = principal quantum number h = Planck's constant m = mass of electron $K = \frac{1}{4\pi\epsilon_0}$ Z = atomic number e = electronic charge
	Velocity of electron	$v = \frac{2\pi K Z e^2}{nh} \Rightarrow v \propto \frac{Z}{n}$	
	Frequency of electron	$\nu = \frac{K Z e^2}{nh r} = \frac{4\pi^2 Z^2 e^4 m K^2}{n^3 h^3}$	
	Kinetic energy (E_k)	$E_k = \frac{2\pi^2 m e^4 Z^2 K^2}{n^2 h^2} = \frac{13.6 Z^2}{n^2} \text{ eV}$	
	Potential energy (E_p)	$E_p = -\frac{4\pi^2 m e^4 Z^2 K^2}{n^2 h^2} = -\frac{27.2 Z^2}{n^2} \text{ eV}$	
	Total energy (E)	$E = -\frac{2\pi^2 m e^4 Z^2 K^2}{n^2 h^2} = -\frac{13.6 Z^2}{n^2} \text{ eV}$ $\Rightarrow E \propto -\frac{Z^2}{n^2}$	

Quantization of Energy (Particle in a Box)

In quantum mechanics, the energies of a system are discrete or quantized. The energy of a particle of mass m confined to a box of length L can have discrete values of energy given by the relation

$$E_n = \frac{n^2 h^2}{8mL^2} ; n = 1, 2, 3, \dots$$



6. HYDROGEN SPECTRUM SERIES

Each element emits a spectrum of radiation, which is characteristic of the element itself. The spectrum consists of a set of isolated parallel lines and is called the line spectrum.

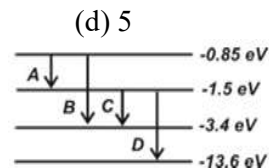
1 Lyman Series (UV region)		When electron jumps from $n = 2, 3, 4, \dots$ orbit to $n = 1$ orbit, then a line of Lyman series is obtained.	Region Ultra violet
2 Balmer Series (Visible region)		When electron jumps from $n = 3, 4, 5, \dots$ orbit to $n = 2$ orbit, then a line of Balmer series is obtained.	Region Visible
3 Paschen Series (IR region)		When electron jumps from $n = 4, 5, 6, \dots$ orbit to $n = 3$ orbit, then a line of Paschen series is obtained.	Region Infrared
4 Brackett Series (IR region)		When electron jumps from $n = 5, 6, 7, \dots$ orbit to $n = 4$ orbit, then a line of Brackett series is obtained.	Region Infrared
5 Pfund Series (IR region)		When electron jumps from $n = 6, 7, 8, \dots$ orbit to $n = 5$ orbit, then a line of Pfund series is obtained.	Region Infrared

★ QUICK REFERENCE CONSTANTS ★

	$e = 1.6 \times 10^{-19} \text{ C}$ (Electronic charge)		$m_e = 9.11 \times 10^{-31} \text{ kg}$ (Mass of electron)		$h = 6.626 \times 10^{-34} \text{ J s}$ (Planck's constant)		$\epsilon_0 = 8.854 \times 10^{-12} \text{ F m}^{-1}$ (Permittivity of free space)		$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$
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MCQ's WITH SOLUTION - ATOMS

- When alpha particles are sent through a thin gold foil, most of them go straight through the foil, because
 (a) α is +ve charged (b) $M_\alpha > M_e$
 (c) Most part of an atom is empty space (d) α moves with high velocity
- In an experiment of scattering of alpha particle showed for the first time that the atom has,
 (a) Electron (b) Proton (c) Neutron (d) Nucleus
- In Geiger Marsden experiment, expression of distance of closest approach to the nucleus of an α particle with KE 'K' before it comes to momentarily at rest & reverse its direction is,
 (a) $\frac{Ze^2}{4\pi\epsilon_0 K}$ (b) $\frac{Ze^2}{2\pi\epsilon_0 K}$ (c) $\frac{Ze^2}{2\epsilon_0 K}$ (d) $\frac{Ze^2}{4\epsilon_0 K}$
- The angular momentum of the electron in the nth allowed orbit is;
 (a) $\frac{Ph}{2\pi}$ (b) $\frac{h}{2\pi}$ (c) $\frac{2h}{\pi}$ (d) $\frac{nh}{2\pi}$
- In equation, what does this $E_n = -\frac{13.6}{n^2} \text{ eV}$, negative sign indicates.
 (a) Electrons are free to move (b) Electron is bound with nucleus.
 (c) Kinetic energy is equal to potential energy (d) Atom is radiating energy
- Kinetic energy of electron in hydrogen atom is
 (a) $\frac{e^2}{4\pi\epsilon_0 r}$ (b) $\frac{e^2}{8\pi\epsilon_0 r}$ (c) $\frac{e^3}{8\pi\epsilon_0 r}$ (d) $\frac{e^2}{3\pi\epsilon_0 r}$
- Energy required to excite an electron in hydrogen atom to its ground state to its first excited state is .
 (a) 6.2 eV (b) 3.40 eV (c) 10.2 eV (d) -13.6 eV
- What is the angular momentum of an electron revolving in the 3rd orbit of an atom?
 (a) $31.5 \times 10^{-34} \text{ J-sec}$ (b) $3.15 \times 10^{-34} \text{ J-sec}$
 (c) $315 \times 10^{-34} \text{ J-sec}$ (d) $0.315 \times 10^{-34} \text{ J-sec}$
- If the electron in hydrogen atoms is excited to $n = 5$ state, the number of different frequencies of radiation which may be emitted is:
 (a) 4 (b) 10 (c) 8 (d) 5
- Energy level diagram of an element is given:- Which transition corresponds to the emission of a spectral line of wave length 102.7 nm
 (a) A (b) B (c) C (d) D
- In Geiger-Marsden scattering experiment, the trajectory traced by an α -particle depends on
 (a) number of collisions (b) number of scattered α -particles
 (c) impact parameter (d) none of these
- The first model of the atom was proposed in 1898 by
 (a) Ernst Rutherford (b) Albert Einstein (c) J. J. Thomson (d) Niels Bohr
- In Bohr's model of the hydrogen atom, the radius of the n^{th} orbit r_n is related to the principal quantum number n as
 (a) $r_n \propto n$ (b) $r_n \propto 1/n$ (c) $r_n \propto n^2$ (d) $r_n \propto 1/n^2$
- Which of the following spectral series of hydrogen atom lies in the visible region of the electromagnetic spectrum?
 (a) Lyman series (b) Balmer series (c) Paschen series (d) Brackett series
- Ground state energy of H- atom is -13.6 eV . Energy required to remove an electron from $n = 2$ is
 (a) 10.2 eV (b) 3.4 eV (c) 6.8 eV (d) 0 eV



ANSWERS

1. c, 2. d, 3. b, 4. d, 5. b, 6. b, 7. c, 8. b, 9. b, 10. d,
11. c, 12. c, 13. c, 14. b, 15. b

ASSERTION AND REASON TYPE QUESTIONS

Directions: These questions consist of two statements, each printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.

- (a) If both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
- (b) If both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
- (c) If the Assertion is correct but Reason is incorrect.
- (d) If both the Assertion and Reason are incorrect.

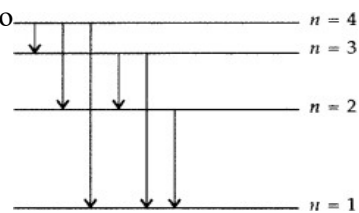
1. **Assertion (A):** According to Bohr's theory in the hydrogen atom, the electron revolves in circular orbits.
Reason (R): The centripetal force is provided by the electrostatic attraction between the proton and the neutron.
2. **Assertion (A):** The frequency of radiation emitted or absorbed by an atom is related to the difference in energy between two energy levels.
Reason (R): The energy of the photon emitted or absorbed is equal to the difference in energy between the two levels.
3. **Assertion (A):** The ground state of an atom is its highest energy state.
Reason (R): In the ground state, the electron occupies the farthest possible orbit to the nucleus.
4. **Assertion (A):** The Bohr model of the atom explains the line spectra of hydrogen atom.
Reason (R): The energy levels of the electron in the hydrogen atom are quantized, leading to discrete spectral lines.
5. **Assertion (A):** The angular momentum of an electron in a Bohr orbit is quantized.
Reason (R): The angular momentum of the electron is an integral multiple of $h/4\pi$.
6. **Assertion (A):** The emission spectrum of an element is characteristic of the element.
Reason (R): The energy levels of electrons in an atom are unique to each element.
7. **Assertion (A):** The ionization energy of an atom is the energy required to remove an electron from the atom in its ground state.
Reason (R): Ionization energy is a measure of the binding energy of the protons in the ground state.
8. **Assertion (A):** The wavelength of light emitted in an electronic transition is inversely proportional to the energy difference between the initial and final states.
Reason (R): The energy of a photon is inversely proportional to its wavelength.
9. **Assertion (A):** The energy levels in an atom are quantized due to the wave nature of electrons.
Reason (R): Electrons exhibit both particle and wave properties
10. **Assertion (A):** Distance of closest approach of α -particle to the nucleus is always greater than the size of the nucleus.
Reason (R): Strong nuclear repulsion does not allow α -particle to reach the surface of nucleus.

ANSWERS

1. c, 2. a, 3. d, 4. a, 5. c, 6. a, 7. c, 8. a, 9. b, 10. d

SHORT ANSWER QUESTIONS: (02 MARKS)

1. What is the shortest wavelength present in Paschen series and Balmer series of H-spectrum?
2. In the Rutherford's scattering experiment the distance of closest approach for an α -particle is d_0 . If α -particle is replaced by a proton, how much kinetic energy in comparison to α particle will it require to have the same distance of closest approach d_0 ?
3. Find the ratio of Bohr's radius in ground state and 1st excited state of H-atom?
4. The value of ground state energy of hydrogen atom is -13.6 eV. (i) what does the negative sign signify? (ii). How much energy is required to take an electron in this atom from the ground state to the first excited state?
5. Use Rydberg formula to determine the wavelength of H_α & H_β line. $R = 1.03 \times 10^7 \text{ m}^{-1}$
6. Calculate the shortest wavelength of the spectral lines emitted in Balmer series.
7. When an electron in hydrogen atom jumps from the third excited state to the ground state, how would the de Broglie wavelength associated with the electron change? Justify your answer.
8. Show that the radius of the orbit in hydrogen atom varies as n^2 , where n is the principal quantum number of the atom.
9. Using Bohr's postulates, obtain the expression for the total energy of the electron in the stationary states of the hydrogen atom. Hence draw the energy level diagram showing how the line spectra corresponding to Balmer series occur due to transition between energy levels.
10. The figure shows energy level diagram of hydrogen atom
 - (a) Find out the transition which results in the emission of a photon of wavelength 496 nm.
 - (b) Which transition corresponds to the emission of radiation of max wavelength? Justify.



SOLUTIONS FOR SHORT ANSWER QUESTIONS: (02 MARKS)

1. Using the Rydberg formula: $\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$,
for Balmer Series $n_f=2$ & $n_i=\infty$,
 $\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right) \Rightarrow \lambda = \frac{4}{R} \Rightarrow 364.6 \text{ nm}$
Similarly for Paschen Series $n_f=3$ & $n_i=\infty$,
 $\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{\infty^2} \right) \Rightarrow \lambda = \frac{9}{R} \Rightarrow 820 \text{ nm}$
2. $K_\alpha = \frac{1}{4\pi\epsilon_0} \cdot \frac{(Ze)(2e)}{d_0}$, $K_p = \frac{1}{4\pi\epsilon_0} \cdot \frac{(Ze)(e)}{d_0} \Rightarrow K_p = \frac{1}{2} K_\alpha$
3. **Ratio of Bohr's Radius in Ground State and First Excited State**
Radius of n th orbit: $r_n = a_0 n^2$ For ground state: $n = 1$, $r_1 = a_0$
For first excited state: $n = 2$, $r_2 = 4a_0$ Therefore, $\frac{r_2}{r_1} = \frac{4a_0}{a_0} = 4$, $r_2:r_1 = 4:1$
4. **Ground State Energy of Hydrogen Atom = -13.6 eV**
(i) **Meaning of Negative Sign** - The negative sign shows that the electron is bound to the nucleus by electrostatic force.
(ii) **Energy Required for First Excited State**
Ground state: $E_1 = -13.6 \text{ eV}$ $\Rightarrow$ First excited state: $E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}$
Required energy: $\Delta E = E_2 - E_1 = (-3.4) - (-13.6) = 10.2 \text{ eV} \Rightarrow \boxed{10.2 \text{ eV}}$

5. Wavelength of H_α and H_β Using Rydberg formula: $\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

(i) H_α line Transition: $n_2 = 3, n_1 = 2$

$$\frac{1}{\lambda} = 1.03 \times 10^7 \left(\frac{1}{4} - \frac{1}{9} \right) = 1.03 \times 10^7 \times \frac{5}{36} \Rightarrow \lambda \approx 6.56 \times 10^{-7} \text{ m.} \Rightarrow \boxed{\lambda_{H_\alpha} = 656 \text{ nm}}$$

(ii) H_β line -Transition: $n_2 = 4, n_1 = 2.$ $\frac{1}{\lambda} = 1.03 \times 10^7 \left(\frac{1}{4} - \frac{1}{16} \right) = 1.03 \times 10^7 \times \frac{3}{16}$

$$\lambda \approx 4.86 \times 10^{-7} \Rightarrow \boxed{\lambda_{H_\beta} = 486 \text{ nm}}$$

6. Shortest Wavelength in Balmer Series

For shortest wavelength: $n_2 = \infty, n_1 = 2 \Rightarrow \frac{1}{\lambda} = R \left(\frac{1}{4} \right)$

$$\lambda = \frac{4}{R} = \frac{4}{10^7} = 4 \times 10^{-7} \text{ m} \Rightarrow \boxed{\lambda = 400 \text{ nm}}$$

7. Change in de Broglie Wavelength

de Broglie relation: $2\pi r = n\lambda \Rightarrow \lambda = \frac{2\pi r}{n} \Rightarrow$ Since: $r_n \propto n^2 \Rightarrow$ therefore $\lambda \propto n$

When electron jumps from $n = 4$ to $n = 1$, the de Broglie wavelength decreases.

8. Show that Radius Varies as n^2

Electrostatic force provides centripetal force: $\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$

Bohr quantization condition: $mv = \frac{nh}{2\pi} \Rightarrow$ Squaring: $m^2 v^2 r^2 = \frac{n^2 h^2}{4\pi^2}$

Using: $mv^2 = \frac{e^2}{4\pi\epsilon_0 r}$ & Substituting, we get $r \propto n^2$

9. Expression for Total Energy of Electron

Electrostatic force: $\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$ & we know Kinetic energy: $K = \frac{1}{2} mv^2$

Using force equation: $K = \frac{e^2}{8\pi\epsilon_0 r}$, Potential energy: $U = -\frac{e^2}{4\pi\epsilon_0 r}$

Total energy: $E = K + U \Rightarrow E = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} \Rightarrow E = -\frac{e^2}{8\pi\epsilon_0 r}$ & hence, $E_n = -\frac{13.6}{n^2} \text{ eV}$

Balmer Series Energy Level Diagram

$n = \infty$ -----
 $n = 5$ ----- ↓
 $n = 4$ ----- ↓
 $n = 3$ ----- ↓
 $n = 2$ ----- ← Balmer Series
 $n = 1$ -----

10. Energy Level Diagram

(a) Transition for 496 nm : 496 nm belongs to Balmer series: $n_2 = 4, n_1 = 2$ i.e. $\boxed{4 \rightarrow 2}$

(b) Transition Corresponding to Maximum Wavelength Using: $E = \frac{hc}{\lambda}$

Maximum wavelength corresponds to minimum energy difference. Hence transition:

$\boxed{4 \rightarrow 3}$ gives maximum wavelength.

Short Answer Type Questions: (03 MARKS)

1. Write two important limitations of Rutherford model which could not explain the observed features of atomic spectra. How were these explained in Bohr's model of hydrogen atom?

Use the Rydberg formula to calculate the wavelength of the H_α line. (Take $R = 1.1 \times 10^7 \text{ m}^{-1}$)

2. Calculate the shortest wavelength in the Balmer series of hydrogen atom. In which region (infra-red, visible, ultraviolet) of hydrogen spectrum does this wavelength lie?
3. Show that the radius of the orbit in hydrogen atom varies as n^2 , where n is the principal quantum number of the atom.
4. An α -particle moving with initial kinetic energy K towards a nucleus of atomic number z approaches a distance 'd' at which it reverses its direction. Obtain the expression for the distance of closest approach 'd' in terms of the kinetic energy of α -particle K .
5. Using Rutherford model of the atom, derive the expression for the total energy of the electron in hydrogen atom. What is the significance of total negative energy possessed by the electron?
6. (i) A hydrogen atom initially in the ground state absorbs a photon which excites it to the $n = 4$ level. Determine the wavelength of the photon.
(ii) The radius of innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. Determine its radius in $n = 4$ orbit.
7. (i) In hydrogen atom, an electron undergoes transition from 2nd excited state to the first excited state and then to the ground state. Identify the spectral series to which these transitions belong.
(ii) Find out the ratio of the wavelengths of the emitted radiations in the two cases.
8. The ground state energy of hydrogen atom is -13.6 eV. What are the kinetic and potential energies of electron in this state?
9. Find the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its second permitted energy level to the first permitted level and the highest permitted energy level to the first permitted level.
10. Calculate the wave number, wavelength and frequency of the spectral line of hydrogen for the transition from $n_2 = 4$ to $n_1 = 2$ (given $R = 1.097 \times 10^7 \text{ m}^{-1}$)

ANSWERS

Q.1 Rutherford Model and H_α Line

Limitations of Rutherford Model

(i) Stability of Atom:- According to Maxwell's electromagnetic theory, an electron revolving around the nucleus should continuously radiate energy. Hence its orbit should shrink and finally the electron should fall into the nucleus. Therefore Rutherford model could not explain stability of atom.

(ii) Atomic Spectra :- Rutherford model could not explain discrete line spectra of hydrogen atom. According to classical theory, atom should emit continuous spectrum.

Explanation by Bohr's Model

(i) Stable Orbits- Electrons revolve only in certain permitted stationary orbits without radiating energy.

(ii) Quantized Energy Levels:- Radiation is emitted or absorbed only when electron jumps between two stationary orbits. Hence discrete spectral lines are obtained.

Calculation of H_α Line : Using Rydberg formula, $\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

For H_α line, $n_1 = 2$, $n_2 = 3 \Rightarrow \frac{1}{\lambda} = 1.1 \times 10^7 \left(\frac{1}{4} - \frac{1}{9} \right) = 1.1 \times 10^7 \times \frac{5}{36} = 1.528 \times 10^6$

$$\lambda = \frac{1}{1.528 \times 10^6} \Rightarrow \boxed{\lambda \approx 6.54 \times 10^{-7} \text{ m}} \text{ or } \boxed{\lambda \approx 654 \text{ nm}}$$

Q.2 Shortest Wavelength in Balmer Series : For shortest wavelength, $n_1 = 2$, $n_2 = \infty$

Using Rydberg formula, $\frac{1}{\lambda} = R \left(\frac{1}{4} \right) \Rightarrow \lambda = \frac{4}{R} = \frac{4}{1.097 \times 10^7}$

$\lambda \approx 3.646 \times 10^{-7} \text{ m}$ or $\lambda \approx 364.6 \text{ nm}$ This wavelength lies in ultraviolet region.

Q.3 Radius of Orbit varies as $n^2 \Rightarrow$ Electrostatic force provides centripetal force, $\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$

Bohr quantization condition, $mv = \frac{nh}{2\pi}$, Squaring, $m^2 v^2 r^2 = \frac{n^2 h^2}{4\pi^2}$ Using, $mv^2 = \frac{e^2}{4\pi\epsilon_0 r}$

Substituting, $r = \frac{\epsilon_0 n^2 h^2}{\pi m e^2}$ Thus, $r_n \propto n^2$ Hence proved.

Q.4 Distance of Closest Approach

At closest approach, entire kinetic energy converts into electrostatic potential energy.

Initial kinetic energy, K Potential energy at distance d , $U = \frac{1}{4\pi\epsilon_0} \frac{(2e)(Ze)}{d}$

Equating energies, $K = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{d} \Rightarrow$ Therefore, $d = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{K}$

Q.5 Total Energy of Electron in Hydrogen Atom

Electrostatic force provides centripetal force, $\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$ Hence, $mv^2 = \frac{e^2}{4\pi\epsilon_0 r}$

Kinetic Energy. $K = \frac{1}{2} mv^2 \Rightarrow K = \frac{e^2}{8\pi\epsilon_0 r}$ & Potential Energy $U = -\frac{e^2}{4\pi\epsilon_0 r}$

Total Energy. $E = K + U \Rightarrow E = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} \Rightarrow E = -\frac{e^2}{8\pi\epsilon_0 r}$

Significance of Negative Energy

Negative sign shows that electron is bound to the nucleus. Energy must be supplied to remove the electron from atom.

Q.6 Wavelength of Photon absorbed Energy of electron in n th orbit, $E_n = -\frac{13.6}{n^2} \text{ eV}$

Ground state $n=1$, $E_1 = -13.6 \text{ eV}$ For $n=4$, $E_4 = \frac{-13.6}{16} = -0.85 \text{ eV}$

Energy absorbed, $\Delta E = E_4 - E_1 = (-0.85) - (-13.6) = 12.75 \text{ eV}$

Using, $\lambda = \frac{hc}{E} \Rightarrow \lambda = \frac{1240}{12.75} \Rightarrow \lambda \approx 97 \text{ nm}$

(i) Radius of $n=4$ Orbit - Given, $r_1 = 5.3 \times 10^{-11} \text{ m}$

Using, $r_n = n^2 r_1 \Rightarrow r_4 = 16 \times 5.3 \times 10^{-11} \Rightarrow r_4 = 8.48 \times 10^{-10} \text{ m}$

Q.7 Spectral Series: (i) Spectral Series - Second excited state, $n=3$, Transition, $3 \rightarrow 2$ belongs to Balmer series.

Transition, $2 \rightarrow 1$, belongs to Lyman series.

(ii) Ratio of Wavelengths. Using, $\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$ For $3 \rightarrow 2$, $\frac{1}{\lambda_1} = R \left(\frac{1}{4} - \frac{1}{9} \right) = \frac{5R}{36}$

For $2 \rightarrow 1$, $\frac{1}{\lambda_2} = R \left(1 - \frac{1}{4} \right) = \frac{3R}{4} \Rightarrow$ Thus, $\frac{\lambda_1}{\lambda_2} = \frac{27}{5}$, hence $\lambda_1 : \lambda_2 = 27 : 5$

Q.8 Kinetic and Potential Energies: Given, $E = -13.6 \text{ eV}$

Relation, $K = -E \Rightarrow K = 13.6 \text{ eV}$ Potential energy, $U = 2E \Rightarrow U = 2(-13.6)$

$\Rightarrow U = -27.2 \text{ eV}$ Thus, $K = 13.6 \text{ eV}$ & $U = -27.2 \text{ eV}$

Q.9 Ratio of Energies of Photons

For transition $2 \rightarrow 1$, $E_2 = -3.4 \text{ eV}$ & $E_1 = -13.6 \text{ eV}$

Photon energy, $\Delta E_1 = 10.2 \text{ eV}$

For transition $\infty \rightarrow 1$, $E_\infty = 0$ & $\Delta E_2 = 13.6 \text{ eV}$ Therefore, $\frac{\Delta E_1}{\Delta E_2} = \frac{10.2}{13.6} = \frac{3}{4}$

Q.10 Wave Number, Wavelength and Frequency: Given, $n_2 = 4$, $n_1 = 2$

Using Rydberg formula, $\bar{\nu} = R \left(\frac{1}{4} - \frac{1}{16} \right) = 1.097 \times 10^7 \times \frac{3}{16} \Rightarrow \boxed{\bar{\nu} = 2.057 \times 10^6 \text{ m}^{-1}}$

Wavelength -- $\lambda = \frac{1}{\bar{\nu}} = \frac{1}{2.057 \times 10^6} \Rightarrow \boxed{\lambda = 4.86 \times 10^{-7} \text{ m}} \text{ or } \boxed{486 \text{ nm}}$

Frequency. Using, $\nu = c\bar{\nu} = 3 \times 10^8 \times 2.057 \times 10^6 \Rightarrow \boxed{\nu = 6.17 \times 10^{14} \text{ Hz}}$

CASE BASED QUESTIONS

(a) *The spectral series of hydrogen atom were accounted for by Bohr using the relation*

$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$, where, R=Rydberg constant = $1.097 \times 10^7 \text{ m}^{-1}$. Lyman series is obtained when an electron jumps to first orbit from any subsequent orbit. Similarly, Balmer series is obtained when an electron jumps to 2nd orbit from any subsequent orbit. Paschen series is obtained when an electron jumps to 3rd orbit from any subsequent orbit. Whereas Lyman series in U.V. region, Balmer series is in visible region and Paschen series lies in infrared region. Series limit is obtained when $n_2 = \infty$.

(i) What is the ratio of minimum to maximum wavelength in Balmer series?

- (a) 5:2 (b) 5:9 (c) 5:8 (d) 9:5

(ii) Which series of hydrogen spectrum can we see through naked eye?

- (a) Laymen (b) Balmer Series (c) paschan (d) none

(iii) What is the wavelength of first spectral line of Lyman series?

- (a) 1215.4 (b) 5121.4 (c) 2115.4 (d) 4211.5

(iv) What is the frequency of first spectral line of Balmer series?

- (a) $4.57 \times 10^{14} \text{ Hz}$ (b) $5.57 \times 10^{14} \text{ Hz}$ (c) $7.57 \times 10^{14} \text{ Hz}$ (d) $0.57 \times 10^{14} \text{ Hz}$

Hz

(b) Lyman series is obtained when an electron jumps to first orbit from any subsequent orbit. Similarly, Balmer series is obtained when an electron jumps to 2nd orbit from any subsequent orbit. Paschen series is obtained when an electron jumps to 3rd orbit from any subsequent orbit. Whereas Lyman series in U.V. region, Balmer series is in visible region and Paschen series lies in infrared region. Series limit is obtained when $n_2 = \infty$.

(i) The wavelength of first spectral line of Lyman series is

- (a) 1215.4 A^0 (b) 1215.4 cm (c) 1215.4 m (d) 1215.4 mm

(ii) The wavelength limit of Lyman series is

- (a) 951.6 A^0 (b) 511.9 A^0 (c) 1215.4 A^0 (d) 911.6 A^0

(iii) The frequency of first spectral line of Balmer series is

- (a) $1.097 \times 10^7 \text{ Hz}$ (b) $4.57 \times 10^{14} \text{ Hz}$ (c) $4.57 \times 10^{15} \text{ Hz}$ (d) $4.57 \times 10^{16} \text{ Hz}$

(iv) Which of the following transitions in hydrogen atom emit photon of highest frequency?

- (a) $n=1$ to $n=2$ (b) $n=2$ to $n=6$ (c) $n=6$ to $n=2$ (d) $n=2$ to $n=1$

(c) Electrons are revolving around the nucleus in particular stable orbits. The energy of the electron is increasing as we go from the orbit closer to nucleus to outer side. The ground state energy is the lowest energy and it is -13.6 eV for hydrogen atom. Thus, the minimum amount of energy required to remove or free the electron from the ground state is the ionisation energy and it has value +13.6 eV. When electrons jumps from higher energy orbit to lower energy orbit emits energy in the form of photons which are in the form of spectral lines and called as emission lines. The light emitted by the ordinary source of light consist

of different wavelength. But the laser light is the monochromatic one which emits light of single wavelength. In case of hydrogen atom, the ground state energy is that energy state for which $n = 1$. And the states for which $n > 1$, all are the excited states. Where n shows the principal quantum number.

- (i) In hydrogen atom, the energy corresponding to principal quantum number $n = 2$ is
 (a) -13.6 eV (b) -3.4 eV (c) $+13.6 \text{ eV}$ (d) $+3.4 \text{ eV}$
- (ii) For ground state of hydrogen atom the value of principal quantum number is
 (a) $n = 2$ (b) $n = 0$ (c) $n = 1$ (d) $n = \text{infinity}$
- (iii) The minimum energy required to remove the electron from the ground state of the hydrogen atom is called as _____
 (a) excitation energy (b) ionisation energy
 (c) ground state energy (d) excited state energy
- (iv) The acronym LASER stands for?
 (a) Light Amplification by Simplification Estimation and Radiation
 (b) Light Amplification by Stimulated Emission of Radiation.
 (c) Light Amplification by Simulated Emission and Radio
 (d) Light Amplifier by Stimulated Emission of Radiation.

ANSWERS

Q.1 : (i) B (ii) B (iii) A (iv) A

Q.2:- (i) A (ii) C (iii) C (iv) D

Q.3:- (i) B (ii) C (iii) C (iv) B

Long Answer Questions

1. (a) Write two important limitations of Rutherford model, which could not explain the observed features of atomic spectra. How were these explained in Bohr's model of Hydrogen atom? Use the Rydberg formula to calculate the wavelength of $H\alpha$ line of the Balmer Series.
 (b) Using Bohr's postulates, obtain the expression for the radius of n^{th} orbit in hydrogen atom.
2. (I) State Bohr's postulates. Using these postulates derive an expression for the total energy of an electron in n^{th} orbit of an atom. What does negative value of this energy signify? What is Bohr's radius?
3. (I) Using Bohr's postulates, derive the expression for the frequency of radiation emitted when electron in hydrogen atom undergoes transition from higher energy state (quantum number n_i) to the lower state (n_f).
 (II) When electron in hydrogen atom jumps from energy state $n_i = 4$ to $n_f = 3, 2, 1$. Identify the spectral series to which the emission lines belong.

ANSWERS

1(a) Limitations of Rutherford Model

1. **Stability of Atom:** According to Rutherford model, electrons revolve around the nucleus in circular orbits. Since a revolving electron is an accelerated charge, it should continuously radiate energy. Therefore: Electron would lose energy continuously, Radius of orbit would decrease, Electron would finally fall into the nucleus. Thus, Rutherford model could not explain the stability of atom.

2. Atomic Spectra

Rutherford model predicted continuous radiation from revolving electrons. Hence it should produce a continuous spectrum. But experimentally, hydrogen atom gives a line spectrum consisting of discrete lines. Therefore, Rutherford model failed to explain atomic spectra.

Bohr's Explanation

1. Stable Orbits: Bohr proposed that electrons revolve only in certain permitted circular orbits called stationary orbits. In these orbits, electrons do not radiate energy, thus atom remains stable.

2. Explanation of Line Spectrum: Radiation is emitted or absorbed only when electron jumps from one orbit to another. $h\nu = E_i - E_f$

Thus only specific frequencies are emitted, giving discrete spectral lines.

Calculation of Wavelength of H α Line

For Balmer series: $n_f = 2$, For H α line: $n_i = 3$, Using Rydberg formula: $\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$

$$\frac{1}{\lambda} = 1.523 \times 10^6 \text{ m}^{-1} \Rightarrow \lambda = 6.56 \times 10^{-7} \text{ m} \Rightarrow \boxed{\lambda = 656 \text{ nm}}$$

1(b) Radius of nth Orbit: According to Bohr model:

Step 1: Electrostatic Force = Centripetal Force $\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r}$

Step 2: Bohr Quantisation Condition $mv = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$

Step 3: Substitute Value of v $\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{m}{r} \left(\frac{nh}{2\pi mr} \right)^2$ on Solving, $r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2}$

For hydrogen atom: $r_n = n^2 a_0$, Where. $a_0 = 0.53 \text{ \AA}$. is called Bohr radius.

Answer - 2. Bohr's Postulates and Total Energy

Bohr's Postulates

1. Electrons revolve in certain stable circular orbits without radiating energy.
2. Only those orbits are allowed for which angular momentum is quantised: $mvr = \frac{nh}{2\pi}$
3. Radiation is emitted or absorbed when electron jumps from one stationary orbit to another. $h\nu = E_i - E_f$

Derivation of Total Energy

Total energy: $E = K + U$, Where K = Kinetic Energy, U = Potential Energy

Step 1: Kinetic Energy $\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r} \Rightarrow mv^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \Rightarrow K = \frac{1}{2} mv^2 = \frac{1}{8\pi\epsilon_0} \frac{e^2}{r}$

Step 2: Potential Energy. $U = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$

Step 3: Total Energy $E = K + U = \frac{1}{8\pi\epsilon_0} \frac{e^2}{r} - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \Rightarrow E = -\frac{1}{8\pi\epsilon_0} \frac{e^2}{r}$

Substituting value of radius, $E_n = -\frac{13.6}{n^2} \text{ eV}$

Significance of Negative Energy

Negative sign indicates that electron is bound to the nucleus. Energy must be supplied to remove electron from atom to infinity.

Bohr Radius :- Radius of first orbit of hydrogen atom is called Bohr radius. $\boxed{a_0 = 0.53 \text{ \AA}}$

Answer : 3. Frequency of Radiation Emitted : When electron jumps from higher orbit (n_i) to lower orbit (n_f): $h\nu = E_i - E_f$

Energy of nth orbit: $E_n = -\frac{13.6}{n^2} \text{ eV} \Rightarrow$ Therefore, $h\nu = 13.6 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \text{ eV}$

Hence,
$$\nu = \frac{13.6}{h} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Identification of Spectral Series

1. Transition: $4 \rightarrow 3$ This belongs to: **Paschen Series**
2. Transition: $4 \rightarrow 2$, This belongs to: **Balmer Series**
3. Transition: $4 \rightarrow 1$ This belongs to: **Lyman Series**

ATOMS

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1. RUTHERFORD SCATTERING EXPERIMENT

Experimental Setup

Observations

- Most α -particles pass straight undeviated.
- Some are deflected through small angles.
- Very few are deflected through large angles.
- A very few (≈ 1 in 8000) rebound back.

Conclusion

Atom has a small, dense, positively charged nucleus at the centre. Most of the space is empty.

2. RUTHERFORD NUCLEAR MODEL OF ATOM

Salient Features

- Atom is mostly empty space.
- Positive charge and almost all the mass concentrated in tiny nucleus.
- Electrons revolve around nucleus.

Limitations

- Accelerating electron should radiate energy and spiral into nucleus.
- Cannot explain stability of atom.
- Cannot explain line spectra.

3. BOHR MODEL & QUANTISATION

Bohr's Postulates

- Electrons revolve around the nucleus in certain stable circular orbits called stationary states without radiating energy.
- Only those orbits are allowed for which angular momentum is quantised.
- Radiation is emitted or absorbed only when electron jumps from one orbit to another.

$$mvr = \frac{nh}{2\pi} \quad (n = 1, 2, 3, \dots)$$

$$h\nu = E_i - E_f$$

Bohr Orbits in Hydrogen

4. HYDROGEN SPECTRUM & ENERGY LEVELS

Energy Level Diagram

Hydrogen Line Spectrum (Emission)

IMPORTANT FORMULAE

A. ELECTROSTATIC & CENTRIPETAL FORCE

Electrostatic force between nucleus (+Ze) and electron (-e) at distance r

$$F = \frac{1}{4\pi\epsilon_0} \frac{(Ze)(-e)}{r^2}$$

For hydrogen (Z = 1)

$$F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

Centripetal force providing condition for circular motion

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

B. BOHR'S QUANTISATION CONDITION

Angular momentum is quantised

$$mvr = \frac{nh}{2\pi} \quad (n = 1, 2, 3, \dots)$$

D. RADIUS OF n^{th} ORBIT

$$r_n = n^2 a_0$$

Bohr radius (for hydrogen)

$$a_0 = 0.53 \text{ \AA} = 0.53 \times 10^{-10} \text{ m}$$

G. RADIATION & SPECTRUM

Frequency of radiation emitted when electron jumps from higher level E_i to lower level E_f

$$h\nu = E_i - E_f$$

Rydberg formula (for hydrogen spectrum)

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (n_i > n_f)$$

R = Rydberg constant = $1.097 \times 10^7 \text{ m}^{-1}$

λ = wavelength of emitted line
R = Rydberg constant
 n_i = Initial orbit number (higher level)
 n_f = Final orbit number (lower level)

C. DE BROGLIE EXPLANATION

Electron behaves as matter wave. Only those orbits are allowed for which a standing wave is formed.

$$2\pi r_n = n\lambda$$

m = mass of electron
 v = speed of electron
 r = radius of orbit
 n = orbit number (1, 2, 3, ...)
 h = Planck's constant
 ϵ_0 = Permittivity of free space
 e = charge of electron

E. VELOCITY OF ELECTRON IN n^{th} ORBIT

$$v_n = \frac{2.18 \times 10^6}{n} \text{ m s}^{-1}$$

F. ENERGY OF ELECTRON IN n^{th} ORBIT

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

ALPHA-PARTICLE SCATTERING RESULTS

1. TRAJECTORIES OF α -PARTICLES

2. NUMBER OF PARTICLES VS SCATTERING ANGLE

3. FAILURE OF RUTHERFORD MODEL

According to classical electrodynamics, revolving electron should radiate energy continuously and spiral into the nucleus.

BOHR ORBIT AS STANDING MATTER WAVE

$$2\pi r_n = n\lambda$$

HYDROGEN SPECTRUM SERIES

Series	Region	Transitions
Lyman Series	(UV)	$n_f = 1, n_i = 2, 3, 4, \dots$ (Transitions ending at $n = 1$)
Balmer Series	(Visible)	$n_f = 2, n_i = 3, 4, 5, \dots$ (Transitions ending at $n = 2$)
Paschen Series	(IR)	$n_f = 3, n_i = 4, 5, 6, \dots$ (Transitions ending at $n = 3$)
Brackett Series	(IR)	$n_f = 4, n_i = 5, 6, 7, \dots$ (Transitions ending at $n = 4$)

ENERGY LEVELS SUMMARY

- Ground state ($n = 1$): Lowest energy, most stable.
- Excited states ($n > 1$): Higher energy levels.
- Ionisation energy (from $n = 1$ to $n = \infty$): 13.6 eV.
- As n increases: $r_n \uparrow, v_n \downarrow, |E_n| \downarrow$

$n \uparrow \rightarrow r \uparrow \rightarrow v \downarrow \rightarrow |E| \downarrow$

QUICK MEMORY TRICKS

- $r_n \propto n^2$ (Radius increases as square of n)
- $v_n \propto 1/n$ (Speed decreases with n)
- $E_n \propto -1/n^2$ (Energy becomes less negative)
- Higher orbit $\rightarrow$ Higher energy $\rightarrow$ Less stable
- Line spectrum $\rightarrow$ Electron jumps between levels

IMPORTANT POINTS

- Atom is mostly empty space.
- Positive charge in tiny nucleus.

BOHR ORBITS ARE STABLE

- Bohr orbits are stable.
- Angular momentum is quantised.

RADIATION IS EMITTED ONLY DURING TRANSITIONS

- Radiation is emitted only during transitions.
- Explains line spectra.

ELECTRON BEHAVES AS MATTER WAVE

- Electron behaves as matter wave.
- Standing wave explains quantisation.

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