

MCQ:- (बहुविकल्पीय प्रश्न) -

Q 1 $\sin^{-1}x + \cos^{-1}x =$; $x \in [-1, 1]$ -

- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) $\frac{3\pi}{2}$

Q 2 $\tan^{-1}x + \cot^{-1}x =$; $x \in \mathbb{R}$

- (a) $\frac{3\pi}{2}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) π

Q 3 $\operatorname{cosec}^{-1}x + \sec^{-1}x =$; $|x| \geq 1$

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) π (d) $\frac{3\pi}{2}$

Q 4 $\tan^{-1}x + \tan^{-1}y =$; $xy < 1$

- (a) $\tan^{-1} \frac{x+y}{1-xy}$ (b) $\tan^{-1} \frac{x-y}{1+xy}$
(c) $\tan^{-1} \frac{x+y}{1+xy}$ (d) None / कोई नहीं

Q 5 $2\tan^{-1}x =$; $-1 < x < 1$

- (a) $\tan^{-1} \frac{2x}{1-x^2}$ (b) $\tan^{-1} \frac{2x}{1+x^2}$
(c) $\tan^{-1} \frac{x}{1-x^2}$ (d) None / कोई नहीं

Q 6 $\cos(\sec^{-1}x + \operatorname{cosec}^{-1}x) =$; $|x| \geq 1$

- (a) $\frac{\pi}{2}$ (b) 0
(c) π (d) None / कोई नहीं

Q 7 $\cot(\tan^{-1}x + \cot^{-1}x) =$?

- (a) 1 (b) $\frac{1}{2}$
(c) 0 (d) ∞

Q 8 $\tan^{-1}(1) + \tan^{-1}\left(\frac{1}{3}\right) =$?

- (a) $\tan^{-1}\left(\frac{4}{3}\right)$ (b) $\tan^{-1}\left(\frac{2}{3}\right)$
(c) $\tan^{-1}(2)$ (d) $\tan^{-1}(3)$

Q 9 $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) =$?

- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

Q 10 $\sin\left(\cos^{-1}\frac{3}{5}\right) =$?

- (a) $\frac{3}{4}$ (b) $\frac{4}{5}$
(c) $\frac{3}{5}$ (d) Not possible / संभव नहीं है।

Q 11 $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right) =$?

- (a) $\frac{8\pi}{3}$ (b) $\frac{\pi}{2}$
(c) $\frac{3\pi}{4}$ (d) π

Q 12 If $\cot^{-1}\left(-\frac{1}{5}\right) = x$ then $\sin x =$?

यदि $\cot^{-1}\left(-\frac{1}{5}\right) = x$, तो $\sin x$ का मान क्या होगा -

- (a) $\frac{1}{\sqrt{26}}$ (b) $\frac{5}{\sqrt{26}}$
(c) $\frac{1}{\sqrt{24}}$ (d) Not possible / संभव नहीं है।

Q 13 If $\cot^{-1}(-\sqrt{3}) = x$, where $x \in [0, \pi]$ then value of x is -

यदि $\cot^{-1}(-\sqrt{3}) = x$ जहाँ $x \in [0, \pi]$ तो x का मान क्या होगा -

- (a) $\frac{5\pi}{6}$ (b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

Q 14 If $\cot^{-1}(-1) = x$, where $x \in [0, \pi]$ then value of x is -

यदि $\cot^{-1}(-1) = x$ जहाँ $x \in [0, \pi]$ तो x का मान क्या होगा -

- (a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$
(c) π (d) $\frac{3\pi}{2}$

Q 15 $\sin(\tan^{-1}x + \cot^{-1}x) =$; ($x \in \mathbb{R}$)
(a) 1 (b) 2
(c) 3 (d) 4

Very Short Question :- (अति लघु उत्तरीय प्रश्न)

2 Marks Question:-

Q 1 Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$, $x > 1$ in the simplest form.

$\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$, $x > 1$ को सरलतम रूप में व्यक्त करें।

Q 2 Prove that, $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

सिद्ध करें कि $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Q 3 Prove that,
 $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$; $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

सिद्ध करें कि,

$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$; $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Q 4 Write in simplest form,

$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$; $0 < x < \pi$
सरलतम रूप में व्यक्त करें

$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$; $0 < x < \pi$

Q 5 Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

$\sin^{-1}\left(-\frac{1}{2}\right)$ का मुख्य मान ज्ञात करें ?

Q 6 Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ का मुख्य मान ज्ञात करें ?

Q 7 If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, then find the value of x .

यदि $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$ तो x का मान ज्ञात करें।

Q 8 Find the value of, $\tan\left[2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right]$

$\tan\left[2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right]$ का मान निकालें।

Q 9 If $\tan^{-1}x + \tan^{-1}3 = \tan^{-1}8$, then find the value of x .

यदि $\tan^{-1}x + \tan^{-1}3 = \tan^{-1}8$ तो x का मान ज्ञात करें।

Short Question :- (लघु उत्तरीय प्रश्न)

3 Marks Question:-

Q 1 Show that, $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

दिखाएँ कि $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Q 2 Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ का मुख्य मान निकालें।

Q 3 Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ का मान ज्ञात करें।

Q 4 Evaluate, $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

हल करें, $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

Q 5. Evaluate, $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

हल करें, $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

Q 6. If $\tan^{-1}\frac{4}{3} = \theta$, then find the value of $\cos \theta$.

यदि $\tan^{-1}\frac{4}{3} = \theta$ तो $\cos \theta$ का मान ज्ञात करें ?

Q 7 Prove that, $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = 3\tan^{-1}\frac{x}{a}$

सिद्ध करें $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = 3\tan^{-1}\frac{x}{a}$

Long Question ; - (दीर्घ उत्तरीय प्रश्न)

5 Marks Question:-

Q 1 Express $\tan^{-1}\frac{\cos x}{1 - \sin x}$; $-\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

सरल रूप में व्यक्त करें,

$\tan^{-1}\frac{\cos x}{1 - \sin x}$; $-\frac{3\pi}{2} < x < \frac{\pi}{2}$

Q 2 Solve $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

हल करें, $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

Q 3 Show that, $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$.

दिखाएं कि, $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$.

Q 4 Evaluate,

$\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

हल करें, $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

Q 5 Evaluate,

$\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

हल करें, $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

Q 6 Find the value of,

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ का मान निकालें।

Q 7 Prove that,

$\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \left(\frac{\pi}{4} - x\right); x < \frac{\pi}{4}$

सिद्ध करें, $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \left(\frac{\pi}{4} - x\right); x < \frac{\pi}{4}$

Q 8 Prove that, $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right) = \left(\frac{\pi}{4} - \frac{x}{2}\right)$

सिद्ध करें, $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right) = \left(\frac{\pi}{4} - \frac{x}{2}\right)$

Q 9 prove that

$\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

[Hint : Let $x = \cos\theta$]

सिद्ध करें, $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

Answer : "Inverse Trigonometric Function Solution"

MCQ 1 - Marks Solution

- 1 - (b) 2 - (c) 3 - (a) 4 - (a) 5 - (a) 6 - (b) 7 - (c) 8 - (c)
9 - (b) 10 - (b) 11 - (d) 12 - (b) 13 - (a) 14 - (b) 15 - (a)

2- Marks Solution

1 Ans :-

$\therefore \cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) : x > 1$

let us suppose,

$x = \sec\theta \Rightarrow \theta = \sec^{-1}x$

$\therefore \cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \cot^{-1}\left(\frac{1}{\sqrt{\sec^2\theta-1}}\right)$

$\Rightarrow \cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \cot^{-1}\left(\frac{1}{\tan\theta}\right)$
 $= \cot^{-1}(\cot\theta)$

$= \theta$

$\Rightarrow \cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \theta = \sec^{-1}x, x > 1$

2 Ans : $L.H.S = \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$= \tan^{-1}\left(\frac{\frac{2}{11} + \frac{7}{24}}{1 - \frac{2}{11} \times \frac{7}{24}}\right)$

$= \tan^{-1}\left(\frac{\frac{48+77}{11 \times 24}}{\frac{264-14}{11 \times 24}}\right)$

$= \tan^{-1}\left(\frac{125}{250}\right)$

$L.H.S = \tan^{-1}\left(\frac{1}{2}\right) = R.H.S$

3 Ans :-

Let us suppose

$$x = \sin \theta \Rightarrow \theta = \sin^{-1} x \quad \dots(1)$$

$$\therefore \text{L.H.S} = 3\sin^{-1} x$$

$$= 3\theta \quad \dots\dots\dots(2)$$

and ,

$$\text{R.H.S} = \sin^{-1}(3x - 4x^3)$$

$$= \sin^{-1}(3\sin \theta - 4\sin^3 \theta)$$

$$= \sin^{-1}(\sin 3\theta)$$

$$\text{R.H.S} = 3\theta \quad \dots\dots\dots(3)$$

From equation (2) and (3) we get,

$$3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$$

4 Ans :-

$$\tan^{-1}\left(\sqrt{\frac{1 - \cos x}{1 + \cos x}}\right) ; 0 < x < \pi$$

$$= \tan^{-1}\left(\sqrt{\frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}}\right)$$

$$= \tan^{-1}\left(\sqrt{\tan^2 \frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\tan \frac{x}{2}\right)$$

$$= \frac{x}{2}$$

5 Ans :-

$$\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin \frac{\pi}{6}\right)$$

$$= \sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$$

$$= -\frac{\pi}{6}$$

$-\frac{\pi}{6}$ be the required principal value as $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

6 Ans :-

$$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \cos^{-1}\left[-\cos\left(\frac{\pi}{4}\right)\right]$$

$$= \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{4}\right)\right]$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4} \in [0, \pi]$$

$\Rightarrow \frac{3\pi}{4}$ be the required principal value .

7 Ans :-

$$\therefore \sin\left(\sin^{-1} \frac{1}{5} + \cos^{-1} x\right) = 1$$

$$\Rightarrow \sin^{-1} \frac{1}{5} + \cos^{-1} x = \sin^{-1} 1$$

$$\Rightarrow \cos^{-1} x = \sin^{-1}(1) - \sin^{-1}\left(\frac{1}{5}\right)$$

$$= \sin^{-1}\left(\sin \frac{\pi}{2}\right) - \sin^{-1} \frac{1}{5}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} \frac{1}{5}$$

$$\Rightarrow \cos^{-1} x = \cos^{-1} \frac{1}{5}$$

$$\Rightarrow x = \frac{1}{5}$$

8 Ans :-

$$\tan\left(2\tan^{-1} \frac{1}{5} - \frac{\pi}{4}\right)$$

$$\text{Let } 2\tan^{-1} \frac{1}{5} = \theta \quad \dots\dots\dots(1)$$

$$\Rightarrow \frac{1}{5} = \tan \frac{\theta}{2}$$

$$\Rightarrow \sin \frac{\theta}{2} = \frac{1}{\sqrt{26}} \text{ \& } \cos \frac{\theta}{2} = \frac{5}{\sqrt{26}}$$

$$\therefore \tan\left(2\tan^{-1} \frac{1}{5} - \frac{\pi}{4}\right)$$

$$= \tan\left(\theta - \frac{\pi}{4}\right)$$

$$= \frac{\tan \theta - \tan \frac{\pi}{4}}{1 + \tan \theta \times \tan \frac{\pi}{4}}$$

$$= \frac{\tan \theta - 1}{1 + \tan \theta} = \frac{\sin \theta - \cos \theta}{\cos \theta + \sin \theta}$$

$$\Rightarrow \tan\left(2\tan^{-1} \frac{1}{5} - \frac{\pi}{4}\right)$$

$$= \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} - \left(\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}\right)}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} + \left(\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}\right)}$$

$$= \frac{2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}} - \left(\frac{25}{26} - \frac{1}{26}\right)}{2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}} + \left(\frac{25}{26} - \frac{1}{26}\right)}$$

$$= \frac{\frac{5}{13} - \frac{12}{13}}{\frac{5}{13} + \frac{5}{13}} = \frac{-\frac{7}{13}}{\frac{10}{13}}$$

$$\Rightarrow \tan\left(2\tan^{-1} \frac{1}{5} - \frac{\pi}{4}\right) = -\frac{7}{17}$$

9 Ans :-

$$\begin{aligned}\therefore \tan^{-1}x + \tan^{-1}3 &= \tan^{-1}8 \\ \Rightarrow \tan^{-1}x &= \tan^{-1}8 - \tan^{-1}3 \\ &= \tan^{-1}\left(\frac{8-3}{1+8 \times 3}\right) \\ &= \tan^{-1}\left(\frac{5}{1+24}\right) = \tan^{-1}\left(\frac{5}{25}\right) \\ \Rightarrow \tan^{-1}x &= \tan^{-1}\frac{1}{5} \\ \therefore x &= \frac{1}{5}\end{aligned}$$

3 Marks Solution

1 Ans :-

$$\begin{aligned}\text{L.H.S} &= 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} \\ &= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} \\ &= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{7}}{1 - \frac{1}{2} \times \frac{1}{7}}\right) \\ &= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{\frac{9}{14}}{\frac{13}{14}}\right) \\ &= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{9}{13} \\ &= \tan^{-1}\left(\frac{\frac{1}{2} + \frac{9}{13}}{1 - \frac{1}{2} \times \frac{9}{13}}\right) \\ &= \tan^{-1}\left(\frac{\frac{31}{26}}{\frac{17}{26}}\right) \\ \Rightarrow \text{L.H.S} &= \tan^{-1}\left(\frac{31}{17}\right) = \text{R.H.S}\end{aligned}$$

2 Ans :-

$$\begin{aligned}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) &= \sin^{-1}\left(\sin\frac{\pi}{4}\right) \\ &= \frac{\pi}{4} \\ \therefore \text{Principal interval of } \sin &\text{ is } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \text{and } \frac{\pi}{4} &\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \Rightarrow \frac{\pi}{4} &\text{ is the principal value of } \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\end{aligned}$$

3 Ans :-

$$\begin{aligned}\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) &= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \sec^{-1}\left(-\sec\frac{\pi}{3}\right) \\ &= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \sec^{-1}\left(\sec\left(\pi - \frac{\pi}{3}\right)\right) \\ &= \frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right) \\ &= \frac{\pi}{3} - \pi + \frac{\pi}{3} \\ &= \frac{2\pi}{3} - \pi = \frac{2\pi - 3\pi}{3} = -\frac{\pi}{3} \\ \Rightarrow \tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) &= -\frac{\pi}{3}\end{aligned}$$

4 Ans :-

$$\begin{aligned}\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) &= \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\sin\frac{\pi}{6}\right)\right) \\ &= \sin\left[\frac{\pi}{3} - \sin^{-1}\left\{\sin\left(-\frac{\pi}{6}\right)\right\}\right] \\ \Rightarrow \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) &= \sin\left(\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right) \\ &= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) \\ &= \sin\left(\frac{\pi}{2}\right) \\ \Rightarrow \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) &= 1\end{aligned}$$

5 Ans :-

$$\begin{aligned}\text{Let } \cos^{-1}\left(-\frac{3}{5}\right) &= \theta \dots\dots\dots(1) \\ \Rightarrow \cos\theta &= -\frac{3}{5} \\ \therefore \sin\theta &= \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} \\ \Rightarrow \sin\theta &= \frac{4}{5} \\ \therefore \sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right] &= \sin(2\theta) \quad [\text{from (1)}] \\ &= 2\sin\theta \cdot \cos\theta \\ &= 2 \times \left(\frac{4}{5}\right) \times \left(-\frac{3}{5}\right) \\ &= -\frac{24}{25} \\ \Rightarrow \sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right] &= -\frac{24}{25}\end{aligned}$$

6 Ans :-

$$\begin{aligned}\therefore \tan^{-1} \frac{4}{3} &= \theta \\ \Rightarrow \tan \theta &= \frac{4}{3} \dots\dots\dots (1) \\ \text{as } 1 + \tan^2 \theta &= \sec^2 \theta \\ \Rightarrow 1 + \left(\frac{4}{3}\right)^2 &= \sec^2 \theta \\ \Rightarrow 1 + \frac{16}{9} &= \sec^2 \theta \\ \Rightarrow \frac{25}{9} &= \sec^2 \theta \\ \therefore \sec \theta &= \pm \frac{5}{3} \\ \Rightarrow \cos \theta &= \frac{1}{\sec \theta} = \frac{1}{\pm \left(\frac{5}{3}\right)} \\ \Rightarrow \cos \theta &= \pm \frac{3}{5}\end{aligned}$$

7 Ans :-

$$\begin{aligned}\text{Let } x &= a \tan \theta \dots\dots\dots (1) \\ \therefore \text{L.H.S} &= \tan^{-1} \left(\frac{3a^2x - x^3}{a^3 - 3ax^2} \right) \\ &= \tan^{-1} \left[\frac{3a^3 \tan \theta - a^3 \tan^3 \theta}{a^3 - 3a^3 \tan^2 \theta} \right] \\ &= \tan^{-1} \left[\frac{a^3 (3 \tan \theta - \tan^3 \theta)}{a^3 (1 - 3 \tan^2 \theta)} \right] \\ &= \tan^{-1} \left[\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right] \\ &= \tan^{-1} [\tan 3\theta] \\ &= 3\theta \\ \text{L.H.S} &= 3\theta = 3 \cdot \tan^{-1} \frac{x}{a} = \text{R.H.S}\end{aligned}$$

5 Marks Solutions

1 Ans :

$$\begin{aligned}\tan^{-1} \left(\frac{\cos x}{1 - \sin x} \right) ; -\frac{3\pi}{2} < x < \frac{\pi}{2} \\ = \tan^{-1} \left(\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{1 - 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}} \right)\end{aligned}$$

$$\begin{aligned}&= \tan^{-1} \left[\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right) \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)}{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}} \right] \\ &= \tan^{-1} \left[\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right) \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)}{\left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)^2} \right] \\ &= \tan^{-1} \left[\frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \right] \\ &= \tan^{-1} \left[\frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}} \right] \\ &= \tan^{-1} \left[\frac{\tan \frac{\pi}{4} + \tan \frac{x}{2}}{1 - \tan \frac{\pi}{4} \cdot \tan \frac{x}{2}} \right] \\ &= \tan^{-1} \left[\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right] \\ \Rightarrow \tan^{-1} \left(\frac{\cos x}{1 - \sin x} \right) &= \frac{\pi}{4} + \frac{x}{2} ; -\frac{3\pi}{2} < x < \frac{\pi}{2}\end{aligned}$$

2 Ans :-

$$\begin{aligned}\therefore \tan^{-1} 2x + \tan^{-1} 3x &= \frac{\pi}{4} \\ \Rightarrow \tan^{-1} \left(\frac{2x + 3x}{1 - 2x \cdot 3x} \right) &= \frac{\pi}{4} \\ \Rightarrow \tan^{-1} \left(\frac{5x}{1 - 6x^2} \right) &= \frac{\pi}{4} \\ \Rightarrow \frac{5x}{1 - 6x^2} &= \tan \frac{\pi}{4} \\ \Rightarrow \frac{5x}{1 - 6x^2} &= 1 \\ \Rightarrow 1 - 6x^2 &= 5x \\ \Rightarrow 6x^2 + 5x - 1 &= 0 \\ \Rightarrow 6x^2 + 6x - x - 1 &= 0 \\ \Rightarrow 6x(x + 1) - (x + 1) &= 0 \\ \Rightarrow (x + 1)(6x - 1) &= 0 \\ \therefore \text{either } x + 1 = 0 &\quad \text{or} \quad 6x - 1 = 0 \\ \Rightarrow x = -1 &\quad \text{or} \quad x = \frac{1}{6}\end{aligned}$$

3 Ans :-

$$\begin{aligned}\text{let } \sin^{-1} \frac{3}{5} &= x \dots\dots\dots (1) \\ \Rightarrow \sin x &= \frac{3}{5} \\ \therefore \cos x &= \sqrt{1 - \sin^2 x}\end{aligned}$$

$$= \sqrt{1 - \frac{9}{25}}$$

$$= \sqrt{\frac{16}{25}}$$

$$\Rightarrow \cos x = \frac{4}{5}$$

$$\text{and } \sin^{-1} \frac{8}{17} = y \dots\dots\dots (2)$$

$$\Rightarrow \sin y = \frac{8}{17}$$

$$\therefore \cos y = \sqrt{1 - \left(\frac{8}{17}\right)^2}$$

$$= \sqrt{\frac{289 - 64}{289}}$$

$$= \sqrt{\frac{225}{289}}$$

$$\Rightarrow \cos y = \frac{15}{17}$$

$$\text{as } \cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$= \frac{4}{5} \times \frac{15}{17} + \frac{3}{5} \times \frac{8}{17}$$

$$= \frac{60}{85} + \frac{24}{85}$$

$$= \frac{60 + 24}{85}$$

$$\Rightarrow \cos(x - y) = \frac{84}{85}$$

$$\Rightarrow x - y = \cos^{-1} \frac{84}{85}$$

$$\text{[by using equation (1) and (2)]}$$

$$\Rightarrow \sin^{-1} \frac{3}{5} - \sin^{-1} \frac{8}{17} = \cos^{-1} \frac{84}{85}$$

4 Ans :

$$\cos \left[\cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) + \frac{\pi}{6} \right]$$

$$\text{we know that principal interval of } \cos \text{ is } [0, \pi]$$

$$\therefore \cos \left[\cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) + \frac{\pi}{6} \right]$$

$$= \cos \left[\cos^{-1} \left(-\cos \frac{\pi}{6} \right) + \frac{\pi}{6} \right]$$

$$= \cos \left[\cos^{-1} \left(\cos \left(\pi - \frac{\pi}{6} \right) \right) + \frac{\pi}{6} \right]$$

$$= \cos \left[\cos^{-1} \left(\cos \frac{5\pi}{6} \right) + \frac{\pi}{6} \right]$$

$$= \cos \left[\frac{5\pi}{6} + \frac{\pi}{6} \right] ; \text{ as } \frac{5\pi}{6} \in [0, \pi]$$

$$\Rightarrow \therefore \cos \left[\cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) + \frac{\pi}{6} \right]$$

$$= \cos \left[\frac{5\pi + \pi}{6} \right]$$

$$= \cos \left(\frac{6\pi}{6} \right)$$

$$= \cos(\pi)$$

$$= -1$$

$$\therefore \cos \left[\cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) + \frac{\pi}{6} \right] = -1$$

5 Ans ; -

$$\sin \left[\frac{\pi}{2} - \sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) \right]$$

$$\text{we know that principal interval of } \sin \text{ is } \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$\therefore \sin \left[\frac{\pi}{2} - \sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) \right]$$

$$= \sin \left[\frac{\pi}{2} - \sin^{-1} \left(-\sin \frac{\pi}{3} \right) \right]$$

$$= \sin \left[\frac{\pi}{2} - \sin^{-1} \left[\sin \left(-\frac{\pi}{3} \right) \right] \right]$$

$$= \sin \left[\frac{\pi}{2} - \left(-\frac{\pi}{3} \right) \right]$$

$$= \sin \left[\frac{\pi}{2} + \frac{\pi}{3} \right]$$

$$= \sin \left[\frac{3\pi + 2\pi}{6} \right]$$

$$= \sin \left[\frac{5\pi}{6} \right]$$

$$= \cos \frac{\pi}{3}$$

$$= \frac{1}{2}$$

$$\therefore \sin \left[\frac{\pi}{2} - \sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) \right] = \frac{1}{2}$$

6 Ans : -

$$\tan^{-1}(1) + \cos^{-1} \left(-\frac{1}{2} \right) + \sin^{-1} \left(-\frac{1}{2} \right) \dots\dots\dots (1)$$

$$\text{we know that principal interval of } \sin \text{ is } \left[-\frac{\pi}{2}, \frac{\pi}{2} \right], \cos \text{ is } [0, \pi] \text{ and } \tan \text{ is } \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) - \left\{ \frac{\pi}{2} \right\}$$

$$\therefore \tan^{-1}(1) = \tan^{-1} \left(\tan \frac{\pi}{4} \right)$$

$$\Rightarrow \tan^{-1} 1 = \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \dots\dots\dots (2)$$

$$\cos^{-1} \left(-\frac{1}{2} \right) = \cos^{-1} \left[-\cos \frac{\pi}{3} \right]$$

$$= \cos^{-1} \left[\cos \left(\pi - \frac{\pi}{3} \right) \right]$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3} \in [0, \pi] \dots\dots\dots(3)$$

And $\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$$= \sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right]$$

$$= -\frac{\pi}{6} \dots\dots\dots(4)$$

by using eq^{ns} (2), (3) and (4)

$$\text{eq}^{\text{iii}}(1) \Rightarrow \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$\Rightarrow \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{3\pi + 8\pi - 2\pi}{12}$$

$$= \frac{9\pi}{12}$$

$$= \frac{3\pi}{4}$$

7 Ans : -

$$\text{L.H.S} = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$$

$$= \tan^{-1}\left[\frac{\cos x(1 - \tan x)}{\cos x(1 + \tan x)}\right]$$

$$= \tan^{-1}\left[\frac{1 - \tan x}{1 + \tan x}\right]$$

$$= \tan^{-1}\left[\frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \times \tan x}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - x\right)\right]$$

$$= \left(\frac{\pi}{4} - x\right) = \text{R.H.S}$$

hence ,

$$\therefore \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \left(\frac{\pi}{4} - x\right)$$

8 Ans: -

$$\text{L.H.S} = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$$

$$= \tan^{-1}\left[\frac{\cos 2 \frac{x}{2}}{1 + \sin 2 \frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{1 + 2\sin \frac{x}{2} \cos \frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)}{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{1 - \tan \frac{x}{2}}{1 + \tan \frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\tan \frac{\pi}{4} - \tan \frac{x}{2}}{1 + \tan \frac{\pi}{4} \times \tan \frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$\Rightarrow \text{L.H.S} = \left(\frac{\pi}{4} - \frac{x}{2}\right) = \text{R.H.S}$$

9 Ans :

$$\text{L.H.S} = \tan^{-1}\left\{\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right\}$$

$$\text{Let } x = \cos \theta \dots\dots\dots(1)$$

$$= \tan^{-1}\left\{\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2\cos^2 \frac{\theta}{2}} - \sqrt{2\sin^2 \frac{\theta}{2}}}{\sqrt{2\cos^2 \frac{\theta}{2}} + \sqrt{2\sin^2 \frac{\theta}{2}}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2}\left(\cos \frac{\theta}{2} - \sin \frac{\theta}{2}\right)}{\sqrt{2}\left(\cos \frac{\theta}{2} + \sin \frac{\theta}{2}\right)}\right\}$$

$$= \tan^{-1}\left\{\frac{1 - \tan \frac{\theta}{2}}{1 + \tan \frac{\theta}{2}}\right\}$$

$$= \tan^{-1}\left\{\frac{\tan \frac{\pi}{4} - \tan \frac{\theta}{2}}{1 + \tan \frac{\pi}{4} \times \tan \frac{\theta}{2}}\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right\}$$

$$= \frac{\pi}{4} - \frac{\theta}{2}$$

$$= \frac{\pi}{4} - \frac{1}{2} \times \cos^{-1} x \text{ [by using eqn(1)]}$$

$$= \text{R.H.S}$$

hence ,

$$\tan^{-1}\left\{\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right\} = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$$