

MCQ:- (बहुविकल्पीय प्रश्न) -

Q 1 The order of the differential equation

$$2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0 \text{ is}$$

अवकल समीकरण $2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0$ की कोटि है:

- (a) 2 (b) 1
(c) 0 (d) Not defined
(अपरिभाषित)

Q 2 The differential equation representing the family of curve $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of

- $y^2 = 2c(x + \sqrt{c})$, जहाँ C एक घनात्मक प्राचल है, से निरूपित होने वाले वक्र-कुल का अवकल समीकरण निम्नांकित में किस प्रकार का होगा?
- (a) order 1, degree 3 (b) order 2, degree 3
(कोटि 1) (घात 3) (कोटि 2) (घात 3)
(c) order 3, degree 3 (d) order 4, degree 4
(कोटि 3) (घात 3) (कोटि 4) (घात 4)

Q 3 A solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2 - x \frac{dy}{dx} + y = 0 \text{ is.}$$

अवकल समीकरण $\left(\frac{dy}{dx}\right)^2 - x \frac{dy}{dx} + y = 0$ का समाधान है:

- (a) $y = 2$ (b) $y = 2x$
(c) $y = 2x - 4$ (d) $y = 2x^2 - 4$

Q 4 The order of the differential equation

$$\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ is}$$

अवकल समीकरण $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ की कोटि है:

- (a) 1 (b) 2
(c) 3 (d) 4

Q 5 The degree of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^2 - 3 \frac{d^2y}{dx^2} + 2 \left(\frac{dy}{dx}\right)^4 = y^4 \text{ is}$$

अवकल समीकरण $\left(\frac{d^3y}{dx^3}\right)^2 - 3 \frac{d^2y}{dx^2} + 2 \left(\frac{dy}{dx}\right)^4 = y^4$ का घात है :

- (a) 4 (b) 2
(c) 3 (d) 1

Q 6 The order and degree of the differential

equation $\frac{d^4y}{dx^4} = y + \left(\frac{dy}{dx}\right)^4$ are respectively

अवकल समीकरण $\frac{d^4y}{dx^4} = y + \left(\frac{dy}{dx}\right)^4$ की कोटि एवं घात क्रमशः है:

- (a) 2,2 (b) 4,1
(c) 2,4 (d) 4,2

Q 7 Integrating factor of the differential equation

$$\frac{dy}{dx} + y = \frac{1+y}{x} \text{ is}$$

अवकल समीकरण $\frac{dy}{dx} + y = \frac{1+y}{x}$ का समाकलन गुणांक है:

- (a) e^x (b) xe^x
(c) e^x/x (d) x/e^x

Q 8 What is the integrating factor of

$$\frac{dy}{dx} + y \sec x = \tan x$$

$\frac{dy}{dx} + y \sec x = \tan x$ का समाकलन गुणांक क्या है?

- (a) $\sec x + \tan x$ (b) $\log(\sec x + \tan x)$
(c) $e^{\sec x}$ (d) $\sec x$

Q 9 The general solution of the differential equation

$$\frac{dy}{dx} = \frac{y}{x} \text{ is}$$

अवकल समीकरण $\frac{dy}{dx} = \frac{y}{x}$ का व्यापक हल है:

- (a) $y = k/x$ (b) $y = kx$
(c) $y = k \log x$ (d) $\log y = kx$

Q 10 If a and b are the order and degree of the differential equation

$$y \frac{dy}{dx} + x^3 \left(\frac{d^2y}{dx^2}\right) + xy = \cos x, \text{ then}$$

यदि a तथा b अवकल समीकरण

$$y \frac{dy}{dx} + x^3 \left(\frac{d^2y}{dx^2}\right) + xy = \cos x$$

के कोटि तथा घात हैं, तो

- (a) $a < b$ (b) $a = b$
 (c) $a > b$ (d) None of these
 (इनमें से कोई नहीं)

Very Short Question (2 Marks)

Q 1 Find the differential equation of the family of curves $x^2 + y^2 = 2ax$.

वक्रों के कुल $x^2 + y^2 = 2ax$ का अवकल समीकरण ज्ञात कीजिए।

Q 2 Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

ऐसे परवलयों के कुल का अवकल समीकरण निर्मित कीजिए, जिनका शीर्ष मूल-बिन्दु पर है और जिनका अक्ष धनात्मक y-अक्ष की दिशा में है।

Solve the following differential equations :-

निम्नलिखित अवकल समीकरणों को हल करें :-

Q 3 $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Q 4 $\frac{dy}{dx} = \sqrt{4-y^2}, -2 < y < 2$

Q 5 $\frac{dy}{dx} = \frac{x-1}{y+2} (y \neq -2)$

Q 6 $\frac{dy}{dx} = 1 - x + y - xy$

Q 7 $\frac{dy}{dx} = (1+x^2)(1+y^2)$

Q 8 $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

Q 9 $x \frac{dy}{dx} - y = x^2$

Q 10 $x \frac{dy}{dx} = x + y$

Short Question (3 Marks)

Solve ; - (हल करें :-)

Q 1 $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

Q 2 $y \log y dx - x dy = 0$

Q 3 $e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$

Q 4 $x dy - y dx = \sqrt{x^2 + y^2} dx$

Q 5 $\frac{dy}{dx} - y = \cos x$

Q 6 $x \frac{dy}{dx} + 2y = x^2$

Q 7 $\frac{dy}{dx} + 2y = \sin x$

Q 8 $\frac{dy}{dx} + \frac{y}{x} = x^2$

Q 9 $\cos^2 x \frac{dy}{dx} + y = \tan x$

Q 10 $(1+x^2)dy + 2xydx = \cot x dx, (x \neq 0)$

Q 11 $(x+y) \frac{dy}{dx} = 1$

Q 12 $x \frac{dy}{dx} = y - x \tan \frac{y}{x}$

Long Question (5 Marks)

Solve (हल करें) :-

Q 1 $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right)dy = 0$

Q 2. $\left(x \cos \frac{y}{x} + y \sin \frac{y}{x}\right) y dx = \left(y \sin \frac{y}{x} - x \cos \frac{y}{x}\right) x dy$

Q 3 $y dx + x \log \left(\frac{y}{x}\right) dy - 2x dy = 0$

Q 4 $(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{2}{x^2 - 1}$

Q 5 $(1+x^2) \frac{dy}{dx} + y = \tan^{-1} x$

Q 6 $\frac{dy}{dx} - 2y = \cos 3x$

Q 7 $(x \log x) \frac{dy}{dx} + y = \frac{2}{x} \log x$

Q 8 $\frac{dy}{dx} - 3y \cot x = \sin 2x$, given that $y=2$, when $x = \frac{\pi}{2}$

Q 9 $(1 + y^2)dx = (\tan^{-1}y - x)dy$

Q 10 $\frac{dy}{dx} + xy = xy^3$

MCQ 1-Marks Solution

Ans : -

1 - (a) 2 - (a) 3 - (c) 4 - (b) 5 - (b) 6 - (b) 7 - (c)

8 - (a) 9 - (b) 10 - (c)

2-Marks Solution

1 Ans : -

Given curve is $x^2 + y^2 = 2ax$ (1)

दिया गया वक्र है $x^2 + y^2 = 2ax$ (1)

on differentiating both sides w.r. to x,

दोनों तरफ x के सापेक्ष अवकलन करने पर,

$$2x + 2y \frac{dy}{dx} = 2a \text{(2)}$$

∴ From (1) and (2)

(1) तथा (2) से ,

$$x^2 + y^2 = x \left(2x + 2y \frac{dy}{dx} \right)$$

$$\text{or, } x^2 + y^2 = 2x^2 + 2xy \frac{dy}{dx}$$

$$\text{or, } 2xy \frac{dy}{dx} + x^2 - y^2 = 0$$

which is the required differential equation .

यही अभीष्ट अवकल समीकरण है।

2 Ans : -

Let P denote the family of parabolas and let (0,a) be the focus of a member of the given family , where a is an arbitrary constant.

माना कि P परवलय के कुल है तथा (0,a) नाभि का नियामक है, जहाँ a कोई स्वेच्छ अचर है।

∴ Equation of family P is

अतः वक्रों के कुल P का समीकरण है

$$x^2 = 4ay \text{(1)}$$

Differentiating both sides of equation (1) w.r. to x, we get,

समीकरण (1) को दोनों तरफ x के सापेक्ष अवकलित करने पर,

$$2x = 4a \frac{dy}{dx}$$

$$\therefore 2x = \frac{x^2}{y} \frac{dy}{dx} \quad \left[\text{from (1), } 4a = \frac{x^2}{y} \right]$$

$$\text{or, } 2y = x \frac{dy}{dx}$$

$$\text{or, } x \frac{dy}{dx} - 2y = 0$$

which is the differential equation of the given family of parabolas.

यही दिये गए परवलयों के कुल का अवकल समीकरण है।

3 Ans : -

$$\text{Given differential equation is } \frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

दिया गया अवकल समीकरण है

$$\text{or, } \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

Integrating both sides, we get

दोनों तरफ समाकलित करने पर,

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\text{or, } \tan^{-1}y = \tan^{-1}x + c$$

$$\text{or, } \tan^{-1}y - \tan^{-1}x = c$$

$$\text{or, } \tan^{-1} \frac{y-x}{1+yx} = c$$

$$\text{or, } \frac{y-x}{1+xy} = \tan c$$

$$\text{or, } \frac{y-x}{1+xy} = k \quad (\text{where } k = \tan c)$$

which is the required general solution.

यही अभीष्ट व्यापक हल है।

4 Ans : -

दिया गया अवकल समीकरण है

$$\text{Given differential equation is } \frac{dy}{dx} = \sqrt{4-y^2}$$

$$\text{or, } \frac{dy}{\sqrt{4-y^2}} = dx$$

on integrating both sides, we get

दोनों तरफ समाकलित करने पर

$$\int \frac{dy}{\sqrt{4-y^2}} = \int dx$$

$$\text{or, } \int \frac{dy}{\sqrt{2^2-y^2}} = \int dx$$

$$\text{or, } \sin^{-1}\left(\frac{y}{2}\right) = x + c$$

$$\text{or, } \frac{y}{2} = \sin(x + c)$$

$$\therefore y = 2\sin(x + c)$$

which is the required general solution.

यही अभीष्ट व्यापक हल है।

5 Ans :-

दिया गया अवकल समीकरण है

$$\text{Given differential equation is } \frac{dy}{dx} = \frac{x-1}{y+2}$$

$$\text{or } (y+2)dy = (x-1)dx$$

दोनों तरफ समाकलित करने पर

integrating both sides, we get

$$\int (y+2)dy = \int (x-1)dx$$

$$\text{or, } \frac{y^2}{2} + 2y = \frac{x^2}{2} - x + c$$

$$\text{or, } \frac{y^2 + 4y}{2} = \frac{x^2 - 2x + 2c}{2}$$

$$y^2 + 4y = x^2 - 2x + k, \text{ where } k = 2c$$

$$y^2 + 4y - x^2 + 2x = k$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

6 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$\frac{dy}{dx} = 1 - x + y - xy$$

$$\text{or, } \frac{dy}{dx} = 1(1-x) + y(1-x)$$

$$\text{or, } \frac{dy}{dx} = (1-x)(1+y)$$

$$\text{or, } \frac{dy}{1+y} = (1-x)dx$$

दोनों तरफ समाकलित करने पर

integrating both sides, we get

$$\int \frac{dy}{1+y} = \int (1-x)dx$$

$$\text{or, } \log|1+y| = x - \frac{x^2}{2} + c$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

7 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$

$$\text{or, } \frac{dy}{1+y^2} = (1+x^2)dx$$

दोनों तरफ समाकलित करने पर

integrating both sides, we get

$$\int \frac{dy}{1+y^2} = \int (1+x^2)dx$$

$$\text{or, } \tan^{-1}y = x + \frac{x^3}{3} + c$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

8 Ans :-

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

$$\text{or, } dy = \left(\frac{1 - \cos x}{1 + \cos x} \right) dx$$

$$\text{or, } dy = \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx$$

$$\text{or, } dy = \tan^2 \frac{x}{2} dx$$

$$\text{or, } dy = \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

दोनों तरफ समाकलित करने पर

integrating both sides, we get

$$\int dy = \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$\text{or, } y = \frac{\tan \frac{x}{2}}{\frac{1}{2}} - x + c$$

$$\text{or, } y = 2\tan \frac{x}{2} - x + c$$

Which is the required general solution

यही अभीष्ट व्यापक हल है।

9 Ans :-

दिया गया अवकल समीकरण है

$$\text{Given differential equation is } x \frac{dy}{dx} - y = x^2$$

$$\text{or } \frac{dy}{dx} - \frac{1}{x} \cdot y = x \dots\dots\dots(1)$$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल

समीकरण है, जहाँ $P = -\frac{1}{x}$, तथा $Q = X$

which is a linear differential equation of the

from $\frac{dy}{dx} + Py = Q$, where $P = -\frac{1}{x}$, $Q = x$

Now I.F. = $e^{\int P dx} = e^{\int -\frac{1}{x} dx} = e^{-\log x} = e^{\log \frac{1}{x}}$

$$\Rightarrow \text{I.F.} = \frac{1}{x}$$

$\therefore$ Solution of the d.e.(1) will be

अतः अवकल समीकरण (1) का हल होगा :-

$$y \cdot \frac{1}{x} = \int \frac{1}{x} \cdot x dx + c$$

$$\text{or, } \frac{y}{x} = x + c \Rightarrow y = x^2 + cx$$

यही अभीष्ट व्यापक हल है।

Which is the required general solution.

10 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is $x \frac{dy}{dx} = x + y$

$$\text{or, } \frac{dy}{dx} = 1 + \frac{y}{x}$$

$$\text{or, } \frac{dy}{dx} - \frac{1}{x} \cdot y = 1 \dots\dots\dots(1)$$

which is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = -\frac{1}{x}, Q = 1$$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल

समीकरण है जहाँ $P = -\frac{1}{x}$, $Q = 1$

$$\text{Now, I.F.} = e^{\int P dx} = e^{\int -\frac{1}{x} dx} = e^{-\log x} = e^{\log \frac{1}{x}} = \frac{1}{x}$$

$\therefore$ Solution of the d.e.(1) will be

अतः अवकल समीकरण (1) का हल होगा :-

$$y \cdot \frac{1}{x} = \int \frac{1}{x} \cdot 1 dx + c$$

$$\text{or, } \frac{y}{x} = \log|x| + c$$

$$\text{or, } y = x \log|x| + cx$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

1 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$$

dividing by $\tan x \cdot \tan y$, we get

$$\frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

समाकलित करने पर

Integrating, we get

$$\int \frac{\sec^2 x}{\tan x} dx + \int \frac{\sec^2 y}{\tan y} dy = \log c$$

$$\text{or, } \log|\tan x| + \log|\tan y| = \log c$$

$$\text{or, } \log|\tan x \cdot \tan y| = \log c$$

$$\text{or, } |\tan x \cdot \tan y| = c$$

$$\therefore \tan x \cdot \tan y = \pm c$$

$$\text{or, } \tan x \cdot \tan y = k \text{ where } k = \pm c$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

2 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is $y \log y dx - x dy = 0$

$$\text{or, } x dy = y \log y dx$$

$$\text{or, } \frac{dy}{y \log y} = \frac{dx}{x} \dots\dots\dots(1)$$

दोनों तरफ समाकलित करने पर

Integrating both sides, we get

$$\int \frac{dy}{y \log y} = \int \frac{dx}{x}$$

$$\text{or, } \log(\log y) = \log|x| + \log|c|$$

$$\Rightarrow \log(\log y) = \log|cx|$$

$$\Rightarrow \log y = cx \Rightarrow y = e^{cx}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

3 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$$

$$\text{or, } \frac{e^x}{1 - e^x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

समाकलित करने पर

Integrating, we get

$$\int \frac{e^x}{1-e^x} dx + \int \frac{\sec^2 y}{\tan y} dy = \log c$$

$$\text{or, } -\log(1-e^x) + \log \tan y = \log c$$

$$\text{or, } \log \frac{\tan y}{1-e^x} = \log c$$

$$\text{or, } \frac{\tan y}{1-e^x} = c \Rightarrow \tan y = c(1-e^x)$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

4 Ans ; -

दिया गया अवकल समीकरण है

Given differential equation is

$$x dy - y dx = \sqrt{x^2 + y^2} dx$$

$$\text{or, } x dy = (y + \sqrt{x^2 + y^2}) dx$$

$$\text{or, } \frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} \dots\dots\dots(1)$$

Which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

$$\text{put } y = vx \quad \therefore \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$\therefore$ Equation (1) becomes

$\therefore$ समीकरण(1) हो जाता है

$$v + x \frac{dv}{dx} = \frac{vx + \sqrt{x^2 + v^2 x^2}}{x}$$

$$\text{or, } x \frac{dv}{dx} = \sqrt{1 + v^2} \quad \text{or, } \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

समाकलित करने पर

Integrating, we get

$$\log |v + \sqrt{1 + v^2}| = \log |x| + \log k$$

$$\Rightarrow |v + \sqrt{1 + v^2}| = k|x|$$

$$\Rightarrow v + \sqrt{1 + v^2} = \pm kx = cx, \text{ where } c = \pm k$$

$$\therefore \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = cx$$

$$\Rightarrow y + \sqrt{x^2 + y^2} = cx^2$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

5 Ans ; -

दिया गया अवकल समीकरण है

$$\text{Given differential equation is } \frac{dy}{dx} - y = \cos x \dots(1)$$

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = -1$, $Q = \cos x$

$$\text{यह } \frac{dy}{dx} + Py = Q \text{ के रूप का रैखिक अवकल}$$

समीकरण है जहाँ $P = -1$, $Q = \cos x$

$$\therefore \text{ I.F} = e^{\int p dx} = e^{\int -1 dx} = e^{-x}$$

$\therefore$ Solution of d.e. (1), will be

अतः अवकल समीकरण (1) का हल होगा :-

$$y.e^{-x} = \int e^{-x} . \cos x dx + c \dots\dots\dots(2)$$

$$\text{Let } I = \int e^{-x} \cos x dx$$

$$= \cos x . (-e^{-x}) - \int -e^{-x} (-\sin x) dx$$

$$= -e^{-x} \cos x - \int e^{-x} \sin x dx$$

$$= -e^{-x} \cos x - \left[\sin x (-e^{-x}) - \int \cos x (-e^{-x}) dx \right]$$

$$= -e^{-x} \cos x + e^{-x} \sin x - \int e^{-x} \cos x dx$$

$$\text{or, } I = e^{-x} (\sin x - \cos x) - I$$

$$\text{or, } 2I = e^{-x} (\sin x - \cos x)$$

$$\therefore I = \frac{e^{-x} (\sin x - \cos x)}{2}$$

$$\therefore \text{ From eqn (2), } y.e^{-x} = \frac{e^{-x} (\sin x - \cos x)}{2} + c$$

$$\therefore y = \frac{\sin x - \cos x}{2} + ce^x$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

6 Ans : -Given differential equation is

दिया गया अवकल समीकरण है

$$x \frac{dy}{dx} + 2y = x^2 \quad \text{or, } \frac{dy}{dx} + \frac{2}{x} y = x \dots\dots(1)$$

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$ where $P = \frac{2}{x}$, $Q = x$

$$\text{यह } \frac{dy}{dx} + Py = Q \text{ के रूप का रैखिक अवकल}$$

समीकरण है, जहाँ $P = \frac{2}{x}$, $Q = x$

$$\therefore \text{ I.F} = e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2$$

$\therefore$ Solution of d.e. (1) will be

$\therefore$ अवकल समीकरण (1) का हल होगा

$$y.x^2 = \int x^2 . x dx + c$$

$$\text{or, } y \cdot x^2 = \frac{x^4}{4} + c$$

$$\therefore y = \frac{x^2}{4} + \frac{c}{x^2}$$

Which is the general solution of given d.e.

यही अभीष्ट व्यापक हल है।

7 Ans :-

दिया गया अवकल समीकरण है

$$\text{Given d.e. is } \frac{dy}{dx} + 2y = \sin x \dots\dots\dots(1)$$

which is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = 2, Q = \sin x$$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल

समीकरण है, जहाँ $P = 2, Q = \sin x$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{\int 2 dx} = e^{2x}$$

$\therefore$ Solution of d.e. (1) will be

$\therefore$ अवकल समीकरण (1) का हल होगा

$$y \cdot e^{2x} = \int e^{2x} \cdot \sin x dx + c \dots\dots\dots(2)$$

$$\text{Let } I = \int e^{2x} \sin x dx$$

$$= \sin x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} \cdot \cos x dx$$

$$= \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\cos x \left(\frac{e^{2x}}{2} \right) - \int \frac{e^{2x}}{2} \cdot (-\sin x) dx \right]$$

$$= \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cdot \cos x}{4} - \frac{1}{4} \int e^{2x} \sin x$$

$$\text{or, } I = \frac{e^{2x} (2 \sin x - \cos x)}{4} - \frac{1}{4} I$$

$$\text{or, } I + \frac{1}{4} I = \frac{e^{2x} (2 \sin x - \cos x)}{4}$$

$$\text{or, } \frac{5}{4} I = \frac{e^{2x} (2 \sin x - \cos x)}{4}$$

$$\therefore I = \frac{e^{2x} (2 \sin x - \cos x)}{5}$$

$\therefore$ From eqn(2)

$\therefore$ समी.(2)से,

$$y \cdot e^{2x} = \frac{e^{2x} (2 \sin x - \cos x)}{5} + c$$

$$\therefore y = \frac{1}{5} (2 \sin x - \cos x) + ce^{-2x}$$

which is the required general solution.

यही अभीष्ट व्यापक हल है।

8 Ans :-

दिया गया अवकल समीकरण है

$$\text{Given d.e. is } \frac{dy}{dx} + \frac{y}{x} = x^2 \dots\dots\dots(1)$$

which is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = \frac{1}{x}, Q = x^2$$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल समीकरण है जहाँ $P = \frac{1}{x}, Q = x^2$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा :-

$$y \cdot x = \int x \cdot x^2 dx + c$$

$$\text{or, } xy = \int x^3 dx + c$$

$$\text{or, } xy = \frac{x^4}{4} + c$$

$$\text{or, } y = \frac{x^3}{4} + \frac{c}{x}$$

which is the required general solution.

यही अभीष्ट व्यापक हल है।

9 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$\cos^2 x \frac{dy}{dx} + y = \tan x$$

$$\text{or, } \frac{dy}{dx} + \sec^2 x \cdot y = \sec^2 x \cdot \tan x \dots\dots\dots(1)$$

which is a linear d.e. of the form $\frac{dy}{dx} + Py = Q$

where $P = \sec^2 x, Q = \sec^2 x \cdot \tan x$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल

समीकरण है, जहाँ $P = \sec^2 x, Q = \sec^2 x \tan x$

$$\text{Now, I.F.} = e^{\int P dx} = e^{\int \sec^2 x dx} = e^{\tan x}$$

$\therefore$ Solution of eqn(1) will be

अतः समीकरण (1) का हल होगा :-

$$y \cdot e^{\tan x} = \int e^{\tan x} \cdot \sec^2 x \cdot \tan x dx + c$$

$$\text{put } \tan x = t \therefore \sec^2 x dx = dt$$

$$\therefore y \cdot e^{\tan x} = \int t \cdot e^t dt + c$$

$$= t \int e^t dt - \int \left(\frac{d(t)}{dt} \int e^t dt \right) dt + c$$

$$\begin{aligned}
&= te^t - \int e^t dt + c \\
&= te^t - e^t + c \\
&= \tan x \cdot e^{\tan x} - e^{\tan x} + c \\
&= e^{\tan x} (\tan x - 1) + c \\
\therefore y &= (\tan x - 1) + ce^{-\tan x}
\end{aligned}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

10 Ans :-

दिया गया अवकल समीकरण है:-

Given differential equation is

$$(1+x^2)dy + 2xydx = \cot x dx$$

$$\text{or, } (1+x^2)\frac{dy}{dx} + 2xy = \cot x$$

$$\text{or, } \frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{\cot x}{1+x^2} \dots\dots\dots(1)$$

which is a linear d.e. of the form $\frac{dy}{dx} + Py = Q$

$$\text{where } P = \frac{2x}{1+x^2}, Q = \frac{\cot x}{1+x^2}$$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल

समीकरण है जहाँ $P = \frac{2x}{1+x^2}, Q = \frac{\cot x}{1+x^2}$

$$\text{Now, I.F.} = e^{\int P dx} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1+x^2$$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा :-

$$y(1+x^2) = \int (1+x^2) \cdot \frac{\cot x}{1+x^2} dx + c$$

$$\text{or, } y(1+x^2) = \int \cot x dx + c$$

$$\text{or, } y(1+x^2) = \log|\sin x| + c$$

$$\therefore y = (1+x^2)^{-1} \log|\sin x| + c(1+x^2)^{-1}$$

Which is required general solution.

यही अभीष्ट व्यापक हल है।

11 Ans :-

$$(x+y)\frac{dy}{dx} = 1, \text{ or, } \frac{dx}{dy} = x+y$$

$$\text{or, } \frac{dx}{dy} - x = y \dots\dots\dots(1)$$

which is a linear d.e. of the form

$$\frac{dx}{dy} + Px = Q, \text{ where, } P = -1, Q = y$$

यह $\frac{dx}{dy} + Px = Q$ के रूप का रैखिक अवकल

समीकरण है जहाँ $P = -1, Q = y$

$$\therefore \text{I.F.} = e^{\int P dy} = e^{\int -1 dy} = e^{-y}$$

$\therefore$ Solution of eqn(1) will be

अतः समीकरण (1) का हल होगा :-

$$x \cdot e^{-y} = \int ye^{-y} dy + c$$

$$\text{or, } x \cdot e^{-y} = -ye^{-y} - \int -e^{-y} dy + c$$

$$= -ye^{-y} - e^{-y} + c = -e^{-y}(1+y) + c$$

$$\therefore x = -(1+y) + ce^y$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

12 Ans :- Given d.e. is $x \frac{dy}{dx} = y - x \tan \frac{y}{x}$

दिया गया अवकल समीकरण है

$$\frac{dy}{dx} = \frac{y}{x} - \tan \frac{y}{x} \dots\dots\dots(1)$$

which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

$\therefore$ Put $y=vx$

$$\therefore \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$\therefore$ Equation (1) becomes

समीकरण (1) हो जाता है

$$v + x \frac{dv}{dx} = v - \tan v$$

$$\Rightarrow x \frac{dv}{dx} = -\tan v$$

$$\text{or, } \frac{dv}{\tan v} = -\frac{dx}{x}$$

$$\Rightarrow \int \frac{dv}{\tan v} = -\int \frac{dx}{x}$$

$$\text{or, } \int \cot v dv = -\log|x| + \log c$$

$$\text{or, } \log(\sin v) + \log|x| = \log c$$

$$\Rightarrow \log|xsinv| = \log c$$

$$\Rightarrow |xsinv| = c$$

$$\Rightarrow xsinv = \pm c = k \text{ (say)}$$

$$\Rightarrow x \sin \frac{y}{x} = k$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

1 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$$

$$\text{or, } \frac{dx}{dy} = -\frac{e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)}{1 + e^{\frac{x}{y}}} \dots\dots\dots(1)$$

which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

$$\text{Put } x = vy, \text{ then } \frac{dx}{dy} = v + y \frac{dv}{dy}$$

∴ From eqn(1),

अतः समीकरण (1) से :-

$$v + y \frac{dv}{dy} = -\frac{e^v(1 - v)}{1 + e^v}$$

$$\begin{aligned} \therefore y \frac{dv}{dy} &= -\frac{e^v(1 - v)}{1 + e^v} - v \\ &= \frac{-e^v(1 - v) - v(1 + e^v)}{1 + e^v} \end{aligned}$$

$$\text{or, } y \frac{dv}{dy} = -\frac{v + e^v}{1 + e^v}$$

$$\text{or, } \frac{1 + e^v}{v + e^v} dv = -\frac{dy}{y}$$

$$\Rightarrow \int \frac{1 + e^v}{v + e^v} dv = -\int \frac{dy}{y}$$

$$\Rightarrow \log|v + e^v| = -\log|y| + c$$

$$\Rightarrow \log|y(v + e^v)| = c$$

$$\Rightarrow |y(v + e^v)| = e^c$$

$$\Rightarrow y\left(\frac{x}{y} + e^{\frac{x}{y}}\right) = \pm e^c = k \text{ (say)}$$

$$\therefore x + ye^{\frac{x}{y}} = k$$

Which is the required general solution .

यही अभीष्ट व्यापक हल है।

2 Ans :-

दिया गया अवकल समीकरण है—

Given differential equation is

$$y\left(x\cos\frac{y}{x} + y\sin\frac{y}{x}\right)dx = x\left(y\sin\frac{y}{x} - x\cos\frac{y}{x}\right)dy$$

$$\text{or, } y\left(x\cos\frac{y}{x} + y\sin\frac{y}{x}\right) = x\left(y\sin\frac{y}{x} - x\cos\frac{y}{x}\right)\frac{dy}{dx}$$

$$\text{or, } \frac{dy}{dx} = \frac{\left\{x\cos\frac{y}{x} + y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\frac{y}{x} - x\cos\frac{y}{x}\right\}x}$$

dividing num. and den. by x^2 , we get

अंश तथा हर को x^2 से विभाजित करने पर,

$$\frac{dy}{dx} = \frac{\left\{\cos\frac{y}{x} + \frac{y}{x}\sin\left(\frac{y}{x}\right)\right\}\frac{y}{x}}{\left\{\frac{y}{x}\sin\frac{y}{x} - \cos\frac{y}{x}\right\}} \dots\dots\dots(1)$$

$$\text{Put } y = vx, \text{ then } \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore \text{From (1), } v + x \frac{dv}{dx} = \frac{(\cos v + v \sin v)v}{v \sin v - \cos v}$$

$$\text{or, } x \frac{dv}{dx} = \frac{v(\cos v + v \sin v)}{v \sin v - \cos v} - v$$

$$\text{or, } x \frac{dv}{dx} = \frac{v(\cos v + v \sin v - v \sin v + \cos v)}{v \sin v - \cos v}$$

$$= \frac{2v \cos v}{v \sin v - \cos v}$$

$$\text{or, } \frac{v \sin v - \cos v}{v \cos v} dv = \frac{2dx}{x}$$

$$\text{or, } \int \frac{v \sin v - \cos v}{v \cos v} dv = 2 \int \frac{1}{x} dx$$

$$\Rightarrow \int \left(\tan v - \frac{1}{v}\right) dv = 2 \log|x| + \log c$$

$$\text{or, } -\log|\cos v| - \log|v| = 2 \log|x| + \log c$$

$$\text{or, } -\log|v \cos v| = 2 \log|x| + \log c$$

$$\text{or, } 2 \log|x| + \log|v \cos v| = -\log c$$

$$\text{or, } \log|x^2 v \cos v| = \log k \quad (-\log c = \log k)$$

$$\Rightarrow |x^2 v \cos v| = k \Rightarrow x^2 v \cos v = \pm k = c$$

$$\text{(Where } \pm k = c)$$

$$\Rightarrow x^2 \frac{y}{x} \cos \frac{y}{x} = c \Rightarrow xy \cos \frac{y}{x} = c$$

Which is the required general solution.
यही अभीष्ट व्यापक हल है।

3 Ans :- दिया गया अवकल समीकरण है

Given differential equation is

$$ydx + x \log\left(\frac{y}{x}\right)dy - 2x dy = 0 \dots\dots\dots(1)$$

$$\text{or, } y \frac{dx}{dy} = 2x - x \log\left(\frac{y}{x}\right)$$

$$\text{or, } \frac{dx}{dy} = 2 \frac{x}{y} - \frac{x}{y} \log\left(\frac{y}{x}\right)$$

$$\text{or, } \frac{dx}{dy} = 2 \cdot \frac{x}{y} + \frac{x}{y} \log\left(\frac{x}{y}\right) \dots\dots\dots(2)$$

$$\left(\because \log a = -\log \frac{1}{a} \right)$$

which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

Put $x = vy$, then $\frac{dx}{dy} = v + y \frac{dv}{dy}$

$\therefore$ From eqn(2), $v + y \frac{dv}{dy} = 2v + v \log v$

$$\Rightarrow y \frac{dv}{dy} = v + v \log v$$

$$\Rightarrow \frac{dv}{v(1 + \log v)} = \frac{dy}{y}$$

$$\Rightarrow \int \frac{dv}{v(1 + \log v)} = \int \frac{dy}{y}$$

$$\Rightarrow \log |1 + \log v| = \log |y| + c$$

$$\Rightarrow \log \left| \frac{1 + \log v}{y} \right| = c$$

$$\Rightarrow \left| \frac{1 + \log v}{y} \right| = e^c$$

$$\therefore \frac{1 + \log v}{y} = \pm e^c = k(\text{say})$$

$$\Rightarrow 1 + \log \frac{x}{y} = ky,$$

Where k is an arbitrary constant.

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

4 Ans :-

दिया गया अवकल समीकरण है

$$\text{Given d.e. is } (x^2 - 1) \frac{dy}{dx} + 2xy = \frac{2}{x^2 - 1}$$

$$\text{or, } \frac{dy}{dx} + \frac{2x}{x^2 - 1} \cdot y = \frac{2}{(x^2 - 1)^2} \dots\dots(1)$$

which is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

$$\text{where } P = \frac{2x}{x^2 - 1}, Q = \frac{2}{(x^2 - 1)^2}$$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल

समीकरण है, जहाँ $P = \frac{2x}{x^2 - 1}, Q = \frac{2}{(x^2 - 1)^2}$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{\int \frac{2x}{x^2 - 1} dx} = e^{\log(x^2 - 1)} = x^2 - 1$$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा :-

$$\begin{aligned} y(x^2 - 1) &= \int (x^2 - 1) \cdot \frac{2}{(x^2 - 1)^2} \cdot dx + c \\ &= \int \frac{2}{x^2 - 1} dx + c \end{aligned}$$

$$\therefore y(x^2 - 1) = 2 \cdot \frac{1}{2} \log \left| \frac{x - 1}{x + 1} \right| + c$$

$$\text{or, } y(x^2 - 1) = \log \left| \frac{x - 1}{x + 1} \right| + c$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

5 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$(1 + x^2) \frac{dy}{dx} + y = \tan^{-1} x$$

$$\text{or, } \frac{dy}{dx} + \frac{1}{1 + x^2} \cdot y = \frac{\tan^{-1} x}{1 + x^2} \dots\dots\dots(1)$$

which is a linear d.e. of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = \frac{1}{1 + x^2}, Q = \frac{\tan^{-1} x}{1 + x^2}$$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल

समीकरण है, जहाँ $P = \frac{1}{1 + x^2}, Q = \frac{\tan^{-1} x}{1 + x^2}$

$$\text{Now, I.F.} = e^{\int P dx} = e^{\int \frac{1}{1 + x^2} dx} = e^{\tan^{-1} x}$$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा :-

$$y \cdot e^{\tan^{-1} x} = \int e^{\tan^{-1} x} \cdot \frac{\tan^{-1} x}{1 + x^2} dx + c$$

$$\text{Put } \tan^{-1} x = t \quad \therefore \frac{1}{1 + x^2} dx = dt$$

$$\therefore y \cdot e^{\tan^{-1} x} = \int t e^t dt + c$$

$$= t \int e^t dt - \int \left\{ \frac{d(t)}{dt} \int e^t dt \right\} dt + c$$

$$= t e^t - \int e^t dt + c$$

$$= t e^t - e^t + c$$

$$\therefore y \cdot e^{\tan^{-1} x} = (t - 1) e^t + c$$

$$\text{or, } y \cdot e^{\tan^{-1} x} = (\tan^{-1} x - 1) e^{\tan^{-1} x} + c$$

$$\therefore y = \tan^{-1} x - 1 + c e^{-\tan^{-1} x}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

6 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$\frac{dy}{dx} - 2y = \cos 3x \dots\dots\dots(1)$$

which is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = -2, Q = \cos 3x$$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल

समीकरण है, जहाँ $P = -2, Q = \cos 3x$

$$\text{Now, I.F.} = e^{\int P dx} = e^{\int -2 dx} = e^{-2x}$$

∴ Solution of the equation (1) will be

अतः समीकरण (1) का हल होगा :-

$$y \cdot e^{-2x} = \int e^{-2x} \cos 3x dx + c \dots\dots\dots(2)$$

$$\text{Let } I = \int e^{-2x} \cos 3x dx$$

then

$$\begin{aligned} I &= \cos 3x \int e^{-2x} dx - \int \left\{ \frac{d}{dx} (\cos 3x) \int e^{-2x} dx \right\} dx \\ &= \frac{-e^{-2x} \cos 3x}{2} - \int (-3 \sin 3x) \left(\frac{-e^{-2x}}{2} \right) dx \\ &= \frac{-e^{-2x} \cos 3x}{2} - \frac{3}{2} \int e^{-2x} \sin 3x dx \\ &= \frac{-e^{-2x} \cos 3x}{2} - \frac{3}{2} \left[-\sin 3x \cdot \frac{e^{-2x}}{2} - \int 3 \cos 3x \cdot \frac{e^{-2x}}{-2} dx \right] \\ &= \frac{-e^{-2x} \cos 3x}{2} + \frac{3e^{-2x} \sin 3x}{4} - \frac{9}{4} \int e^{-2x} \cos 3x dx \end{aligned}$$

$$\text{or, } I = \frac{-2e^{-2x} \cos 3x + 3e^{-2x} \sin 3x}{4} - \frac{9}{4} I$$

$$\text{or, } I + \frac{9}{4} I = \frac{e^{-2x} (-2 \cos 3x + 3 \sin 3x)}{4}$$

$$\text{or, } \frac{13}{4} I = \frac{e^{-2x} (-2 \cos 3x + 3 \sin 3x)}{4}$$

$$\Rightarrow I = \frac{1}{13} e^{-2x} (-2 \cos 3x + 3 \sin 3x)$$

∴ From eqn(2),

$$y \cdot e^{-2x} = \frac{e^{-2x}}{13} (3 \sin 3x - 2 \cos 3x) + c$$

$$\Rightarrow y = \frac{1}{13} (3 \sin 3x - 2 \cos 3x) + ce^{2x}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

7 Ans :-

दिया गया अवकल समीकरण है

$$\text{Given d.e. is } x \log x \cdot \frac{dy}{dx} + y = \frac{2}{x} \log x$$

$$\text{or, } \frac{dy}{dx} + \frac{1}{x \log x} \cdot y = \frac{2}{x^2} \dots\dots\dots(1)$$

which is a linear d.e. of the form $\frac{dy}{dx} + Py = Q$

$$\text{where } P = \frac{1}{x \log x} \quad \text{and} \quad Q = \frac{2}{x^2}$$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल

समीकरण है, जहाँ $P = \frac{1}{x \log x}$ and $Q = \frac{2}{x^2}$

$$\text{Now, I.F.} = e^{\int P dx} = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$$

∴ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा :-

$$\begin{aligned} y \cdot \log x &= \int \frac{2}{x^2} \cdot \log x dx + c \\ &= 2 \left[\log x \int \frac{1}{x^2} dx - \int \left\{ \frac{d(\log x)}{dx} \int \frac{1}{x^2} dx \right\} dx \right] + C \\ &= 2 \left[\log x \left(-\frac{1}{x} \right) - \int \frac{1}{x} \left(-\frac{1}{x} \right) dx \right] + c \\ &= -\frac{2 \log x}{x} + 2 \int \frac{1}{x^2} dx + c \\ &= -\frac{2 \log x}{x} - \frac{2}{x} + c \\ &= -\frac{2}{x} (\log x + 1) + c \\ \therefore y \cdot \log x &= -\frac{2}{x} (\log x + 1) + c \end{aligned}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

8 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$\frac{dy}{dx} - 3 \cot x \cdot y = \sin 2x \dots\dots\dots(1)$$

which is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = -3 \cot x, Q = \sin 2x$$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल

समीकरण है, जहाँ $P = -3 \cot x$ and $Q = \sin 2x$

$$\text{Now, I.F.} = e^{\int P dx} = e^{\int -3 \cot x dx} = e^{-3 \log \sin x} = e^{\log(\sin x)^{-3}}$$

$$= (\sin x)^{-3} = \frac{1}{\sin^3 x}$$

∴ Solution of the given differential equation(1) will be

अतः दिए गए अवकल समीकरण (1) का हल होगा :-

$$\begin{aligned} y \times \frac{1}{\sin^3 x} &= \int \left(\sin 2x \times \frac{1}{\sin^3 x} \right) dx + c \\ \text{or, } \frac{y}{\sin^3 x} &= \int \frac{2 \sin x \cdot \cos x}{\sin^3 x} dx + c \\ &= 2 \int \frac{\cos x}{\sin^2 x} dx + c \end{aligned}$$

$$= 2 \int \frac{dt}{t^2} + c \quad \text{where } \sin x = t$$

$$= -\frac{2}{t} + c = -\frac{2}{\sin x} + c$$

$$\therefore \frac{y}{\sin^3 x} = -\frac{2}{\sin x} + c$$

$$\therefore y = -2\sin^2 x + c\sin^3 x \dots\dots\dots(2)$$

$$\text{Now, given } y = 2 \text{ when } x = \frac{\pi}{2}$$

$$\therefore \text{From eqn(2), } 2 = -2 + c \Rightarrow c = 4$$

$$\therefore y = -2\sin^2 x + 4\sin^3 x$$

Which is the required particular solution.

यही अभीष्ट विशेष हल है।

9 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$(1 + y^2)dx = (\tan^{-1} y - x)dy$$

$$\text{or, } \frac{dx}{dy} = \frac{\tan^{-1} y - x}{1 + y^2}$$

$$\text{or, } \frac{dx}{dy} = \frac{\tan^{-1} y}{1 + y^2} - \frac{1}{1 + y^2} \cdot x$$

$$\text{or, } \frac{dx}{dy} + \frac{1}{1 + y^2} \cdot x = \frac{\tan^{-1} y}{1 + y^2} \dots\dots\dots(1)$$

which is a linear diff. equation of the form

$$\frac{dx}{dy} + Px = Q, \text{ where } P = \frac{1}{1 + y^2}, Q = \frac{\tan^{-1} y}{1 + y^2}$$

यह $\frac{dx}{dy} + Px = Q$ के रूप का रैखिक अवकल

समीकरण है, जहाँ $P = \frac{1}{1 + y^2}, Q = \frac{\tan^{-1} y}{1 + y^2}$

$$\therefore \text{I.F.} = e^{\int P dy} = e^{\int \frac{1}{1 + y^2} dy} = e^{\tan^{-1} y}$$

$\therefore$ Solution of the differential equation (1) will be

अतः अवकल समीकरण (1) का हल होगा :-

$$x \cdot e^{\tan^{-1} y} = \int e^{\tan^{-1} y} \cdot \frac{\tan^{-1} y}{1 + y^2} dy + c$$

$$= \int te^t dt + c, \text{ where } \tan^{-1} y = t$$

$$= t \int e^t dt - \int \left\{ \frac{d(t)}{dt} \cdot \int e^t dt \right\} dt + c$$

$$= te^t - \int e^t dt + c$$

$$= te^t - e^t + c$$

$$= e^t(t - 1) + c$$

$$\therefore x \cdot e^{\tan^{-1} y} = e^{\tan^{-1} y}(\tan^{-1} y - 1) + c$$

$$\therefore x = \tan^{-1} y - 1 + ce^{-\tan^{-1} y}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

10 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$$\frac{dy}{dx} + xy = xy^3 \dots\dots\dots(1)$$

$$\text{or, } \frac{1}{y^3} \cdot \frac{dy}{dx} + \frac{xy}{y^3} = x$$

$$\text{or, } \frac{1}{y^3} \frac{dy}{dx} + \frac{1}{y^2} \cdot x = x \dots\dots\dots(2)$$

$$\text{Put } \frac{1}{y^2} = t \quad \therefore -\frac{2}{y^3} \frac{dy}{dx} = \frac{dt}{dx}$$

$$\text{or, } \frac{1}{y^3} \frac{dy}{dx} = -\frac{1}{2} \frac{dt}{dx} \dots\dots\dots(3)$$

$\therefore$ Equation (2) becomes

$$-\frac{1}{2} \frac{dt}{dx} + t \cdot x = x$$

$$\text{or, } \frac{dt}{dx} - 2x \cdot t = -2x \dots\dots\dots(4)$$

which is a linear differential equation of the form

$$\frac{dt}{dx} + Pt = Q, \text{ where } P = -2x \text{ and } Q = -2x$$

यह $\frac{dt}{dx} + Pt = Q$ के रूप का रैखिक अवकल

समीकरण है जहाँ $P = -2x$ and $Q = -2x$

$$\therefore \text{I.F.} = e^{\int P dx} = e^{\int -2x dx} = e^{-x^2}$$

$\therefore$ Solution of equation (4) will be

अतः समीकरण (4) का हल होगा :-

$$t \cdot e^{-x^2} = \int e^{-x^2} (-2x) dx + c$$

$$= e^{-x^2} + c$$

$$\Rightarrow \frac{1}{y^2} \cdot e^{-x^2} = e^{-x^2} + c$$

$$\Rightarrow \frac{1}{y^2} = 1 + ce^{x^2}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।