अध्याय - 12

Limits And Derivatives
सीमा और अवकलन

Limits(सीमा)

Limit :- We say that $\lim_{x\to a} f(x) = l$ if whenever

$x \to a$, $f(x) \to l$.

Fundamental Theorems on Limits

1. $\lim_{x\to a} \{f(x) + g(x)\} = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a} \{f(x) - g(x)\} = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a} \{f(x)\cdot g(x)\} = \lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x)$
4. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, $\lim_{x\to a} g(x) \neq 0$
5. $\lim_{x\to a} \{c\cdot f(x)\} = c\cdot \lim_{x\to a} f(x)$, [c= constant]
6. $\lim_{x\to a} c = c$
7. If $f(x) \le g(x)$ for all x then $\lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$

   सभी x के लिए यदि $f(x) \le g(x)$ है तो

   $\lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$

Remember(याद करें)

1. $\lim_{x\to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$, where $a > 0$
2. $\lim_{x\to 0} \frac{a^x - 1}{x} = \log a$
3. $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$
4. $\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$
5. $\lim_{x\to 0} (1+x)^{1/x} = e$
6. $\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{x}{\sin x} = 1$
7. $\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{x}{\tan x} = 1$
8. $\lim_{x\to 0} \cos x = 1$
9. $\lim_{x\to 0} \sin x = 0$

Multiple Choice Questions
(बहु विकल्पीय प्रश्न)

1. The value of $\lim_{x\to 0} 3x$ is

$\lim_{x\to 0} 3x$ का मान है

(a) 3 (b) $\frac{1}{3}$

(c) 1 (d) 0

2. The value of $\lim_{x\to 2} (3x+5)$ is

(a) 3 (b) 5

(c) 11 (d) -11

3. The value of $\lim_{r\to 1} \pi r^2$ is

(a) π (b) 1

(c) 0 (d) None of these

4. The value of $\lim_{x\to 0} (2x - 1)$ is

(a) 2 (b) -1

(c) 1 (d) 0

5. The value of $\lim_{x\to 1} (x^2 + 2x + 1)$ is

(a) 4 (b) -4

(c) 1 (d) 2

6. The value of $\lim_{x\to 3} (x^2 - 9)$ is

(a) 9 (b) 0

(c) 4 (d) -9

7. The value of $\lim_{x\to 0} \frac{x+5}{x-1}$ is

(a) -5 (b) 3

(c) 5 (d) 0

8. The value of $\lim_{x\to -2} (x^2 - 4x)$ is

(a) -4 (b) 12

(c) -12 (d) 4

9. The value of $\lim_{x\to 1} (x+1)$ is

(a) 1 (b) 2

(c) 3 (d) 4

10. The value of $\lim_{x\to 5} (2 - x^2)$ is

(a) 23 (b) 20

(c) -23 (d) 12

11. $\lim_{x\to 0} \log(1 - x)$ is equal to

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) None of these

12. $\lim_{x\to 5} \log(2 + x)$ is equal to

(a) 0 (b) log2

(c) log5 (d) log7

13. $\lim_{x\to 0} (e^x + 2)$ is equal to

(a) 0 (b) 2

(c) 3 (d) 1

14. $\lim_{x\to 0}(e^x + e^{2x} + 2x)$ **is equal to**

(a) 0 (b) 2
(c) 3 (d) 1

15. $\lim_{x\to 0}(1 + x + x^2 + x^3 + x^4)$ **is equal to**

(a) 0 (b) 1
(c) 5 (d) 4

16. $\lim_{x\to -1}(1 + x + x^2 + \ldots\ldots\ldots + x^{10})$ **is equal to**

(a) 0 (b) 1
(c) -1 (d) 2

17. **If** $f(x) = \frac{x+2}{x+1}$, $x \neq -1$ **then** $\lim_{x\to 0} f(x)$ **is**

यदि $f(x) = \frac{x+2}{x+1}$, $x \neq -1$ **तो** $\lim_{x\to 0} f(x)$ **का मान है**

(a) 1 (b) 2
(c) -2 (d) 3

18. **If** $f(x) = e^{2x} - 8$ **then** $\lim_{x\to 0} f(x)$ **is**

(a) -8 (b) -7
(c) 8 (d) 7

19. **If** $f(x) = \log(x+1) + 2$ **, then** $\lim_{x\to 0} f(x)$ **is**

(a) 3 (b) 2
(c) 0 (d) 1

20. **The value of** $\lim_{x\to 3}\left(\frac{x^2+9}{x+3}\right)$ **is**

(a) 18 (b) 6
(c) 3 (d) 2

21. **The value of** $\lim_{x\to 2}\left(\frac{x^2-4}{x-2}\right)$ **is**

(a) 4 (b) 2
(c) 8 (d) 0

22. **The value of** $\lim_{x\to 3}\left(\frac{x^4-81}{x-3}\right)$ **is**

(a) 108 (b) 100
(c) 81 (d) 0

23. **The value of** $\lim_{x\to a}\left(\frac{x^{5/2}-a^{5/2}}{x-a}\right)$ **is**

(a) $\frac{5}{2}$ (b) $\frac{5a^{3/2}}{2}$
(c) $\frac{2a^{3/2}}{5}$ (d) $a^{3/2}$

24. $\lim_{x\to 4}\left(\frac{x^3-64}{x^2-16}\right)$ **is equal to**

(a) 6 (b) -6
(c) 0 (d) 2

25. $\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)$ **is equal to**

(a) -1 (b) 0
(c) 1 (d) None of these

26. **The value of** $\lim_{x\to 0}\left(\frac{e^{2x}-1}{x}\right)$ **is**

(a) 1 (b) 2
(c) 0 (d) 3

27. **The value of** $\lim_{x\to 0}\left(\frac{e^{5x}-1}{10x}\right)$ **is**

(a) 5 (b) $\frac{1}{2}$
(c) 2 (d) $\frac{1}{5}$

28. **The value of** $\lim_{x\to 0}\left(\frac{e^{mx}-1}{nx}\right)$ **is**

(a) $\frac{m}{n}$ (b) m
(c) n *(d)* $\frac{1}{m}$

29. **The value of** $\lim_{x\to 0}\left(\frac{e^{3x}-e^{2x}}{x}\right)$ **is**

(a) 1 (b) 3
(c) 5 (d) 0

30. **The value of** $\lim_{x\to 0}\left(\frac{e^{mx}-e^{nx}}{x}\right)$ **is**

(a) $m-n$ (b) $m+n$
(c) mn (d) $\frac{m}{n}$

31. $\lim_{x\to 0}\left(\frac{e^{5x}-e^{2x}}{6x}\right)$ **is equal to**

(a) $\frac{1}{2}$ (b) 3
(c) 2 (d) $\frac{1}{3}$

32. $\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)$ **is equal to**

(a) 3 (b) log3
(c) 1 (d) log2

33. $\lim_{x\to 0}\left(\frac{5^x-1}{2x}\right)$ **is equal to**

(a) log5 (b) $\frac{1}{2}$log5
(c) $\frac{1}{2}$ (d) log2

34. $\lim_{x\to 0}\left(\frac{a^x-b^x}{x}\right)$ **is equal to**

(a) log$a$ + log$b$ (b) log$a$ - log$b$
(c) log(a - b) (d) log(a + b)

35. $\lim_{x\to 0}\frac{5^x-2^x}{x}$ **is equal to**

(a) log5 + log2 (b) log5 - log2
(c) log(3) (d) log(7)

36. **The value of** $\lim_{x\to 0}\frac{2^x-1}{2x}$ **is**

(a) $\frac{1}{2}$ (b) $\log 2$

(c) $\frac{1}{2}\log 2$ (d) $-\frac{1}{2}\log 2$

37. **The value of** $\lim_{x\to 0}\frac{\log(1+x)}{x}$ **is**

(a) 0 (b) 2

(c) 3 (d) 1

38. **The value of** $\lim_{x\to 0}\frac{\log(1+3x)}{x}$ **is**

(a) 0 (b) 2

(c) 3 (d) 1

39. **If** $f(x)=\frac{\log(1+2x)}{x}$ **then** $\lim_{x\to 0} f(x)$ **is**

(a) 0 (b) 2

(c) 3 (d) 1

40. **If** $f(x)=\frac{\log(1+5x)}{10x}$ **then** $\lim_{x\to 0} f(x)$ **is**

(a) 5 (b) $\frac{1}{2}$

(c) 2 (d) 10

41. **The value of** $\lim_{x\to 0}\left(\frac{\sin 2x}{x}\right)$ **is**

(a) 1 (b) 0

(c) 2 (d) $\frac{1}{2}$

42. **The value of** $\lim_{x\to 0}\left(\frac{\tan 5x}{x}\right)$ **is**

(a) 1 (b) 4

(c) 5 (d) 0

43. **If** $f(x)=\left(\frac{\sin 2x}{4x}\right)$ **then** $\lim_{x\to 0} f(x)$ **is**

(a) 2 (b) $\frac{1}{2}$

(c) 4 (d) 1

44. $\lim_{x\to 0}\left(\frac{\sin 4x}{2x}\right)$ **is equal to**

(a) 2 (b) 4

(c) 8 (d) 6

45. $\lim_{x\to 0}\left(\frac{tan3x}{6x}\right)$ **is equal to**

(a) 2 (b) 3

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

46. **The value of** $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)$ **is**

(a) 4 (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 2

47. **The value of** $\lim_{x\to 0}\frac{\sin mx}{\sin nx}$ **is**

(a) m (b) $m+n$

(c) $\frac{m}{n}$ (d) mn

48. **The value of** $\lim_{x\to 0}\frac{\tan 9x}{3x}$ **is**

(a) 9 (b) 6

(c) 12 (d) 3

49. $\lim_{x\to 0}\frac{\sin 5x}{\tan 6x}$ **is equal to**

(a) $\frac{5}{6}$ (b) $\frac{6}{5}$

(c) 30 (d) 11

50. $\lim_{x\to 0}\frac{\tan 8x}{\sin 4x}$ **is equal to**

(a) 8 (b) 4

(c) $\frac{1}{2}$ (d) 2

51. **The value of** $\lim_{x\to 0}\frac{\tan 7x}{\tan 14x}$ **is**

(a) 7 (b) 14

(c) $\frac{1}{2}$ (d) 2

52. **The value of** $\lim_{x\to 0}(1+\cos x)$ **is**

(a) 2 (b) 1

(c) 0 (d) None of these

53. **The value of** $\lim_{x\to 0}(2+\sin x)$ **is**

(a) 2 (b) 1

(c) 0 (d) None of these

54. $\lim_{x\to 0}(\sin x+\cos x)$ **is equal to**

(a) 0 (b) 1

(c) 2 (d) 3

55. $\lim_{x\to 0}(x+\sin x)$ **is equal to**

(a) 0 (b) 1

(c) 2 (d) 3

56. $\lim_{x\to 0}\sqrt{x+4}$ **is equal to**

(a) 0 (b) 1

(c) 2 (d) 3

57. $\lim_{x\to 0}\sqrt{\frac{x+4}{x+1}}$ **is equal to**

(a) 2 (b) 1

(c) 4 (d) 0

58. **The value of** $\lim_{x\to 2}\frac{x^2+2x+2}{5x}$ **is**

(a) 2 (b) 1

(c) 4 (d) 0

59. **The value of** $\lim_{x\to 0}\frac{\sin^2 mx}{\sin^2 nx}$ **is**

(a) $\frac{m^2}{n^2}$ (b) $\frac{m}{n}$

(c) mn *(d)* m^2+n^2

60. The value of $\lim_{x\to 0}\frac{\sin^2 3x}{\sin^2 2x}$ is

(a) $\frac{3}{2}$ (b) $\frac{9}{4}$
(c) 5 (d) 36

61. The value of $\lim_{x\to 0}\frac{\tan^2 6x}{\tan^2 3x}$ is

(a) 36 (b) 9
(c) 18 (d) 4

62. The value of $\lim_{x\to 0}\frac{\sin^2 5x}{\tan^2 4x}$ is

(a) 100 (b) $\frac{25}{16}$
(c) $\frac{16}{25}$ (d) $\frac{5}{4}$

63. If $f(x) = \sin x$ then $\lim_{x\to\frac{\pi}{2}} f(x)$ is

(a) 0 (b) 2
(c) 3 (d) 1

64. $\lim_{x\to\pi}(\sin x+\cos x)$ is equal to

(a) 1 (b) -1
(c) 0 (d) 2

65. $\lim_{x\to\frac{\pi}{6}}(\sin x-\cos x)$ is equal to

(a) $\frac{1-\sqrt{3}}{2}$ (b) $\frac{1+\sqrt{3}}{2}$
(c) $1+\sqrt{3}$ (d) $1-\sqrt{3}$

66. The value of $\lim_{x\to\frac{\pi}{4}}(\tan x)$ is

(a) -1 (b) 0
(c) 1 (d) None of these

67. The value of $\lim_{x\to 0}(x\cdot\cot 2x)$ is

(a) 1 (b) -1
(c) $\frac{1}{2}$ (d) 2

68. If $f(x) = \tan x+\cot x$ then $\lim_{x\to\frac{\pi}{4}} f(x)$ is

(a) 2 (b) 1
(c) 0 (d) -1

69. If $f(x) = \sqrt{x^2+4x+1}$ then $\lim_{x\to 0} f(x)$ is

(a) 0 (b) 1
(c) -1 (d) 2

70. $\lim_{x\to 2}\left(\frac{x^2+4}{x+2}\right)$ is equal to

(a) 8 (b) 4
(c) 1 (d) 2

Answer key उत्तरमाला

Multiple Choice Questions
(बहु विकल्पीय प्रश्न)

(1)	d	(2)	c	(3)	a	(4)	b	(5)	a
(6)	b	(7)	a	(8)	b	(9)	b	(10)	c
(11)	a	(12)	d	(13)	c	(14)	b	(15)	b
(16)	b	(17)	b	(18)	b	(19)	b	(20)	c
(21)	a	(22)	a	(23)	b	(24)	a	(25)	c
(26)	b	(27)	b	(28)	a	(29)	a	(30)	a
(31)	a	(32)	b	(33)	b	(34)	b	(35)	b
(36)	c	(37)	d	(38)	c	(39)	b	(40)	b
(41)	c	(42)	c	(43)	b	(44)	a	(45)	c
(46)	d	(47)	c	(48)	d	(49)	a	(50)	d
(51)	c	(52)	a	(53)	a	(54)	b	(55)	a
(56)	c	(57)	a	(58)	b	(59)	a	(60)	b
(61)	d	(62)	b	(63)	d	(64)	b	(65)	a
(66)	c	(67)	c	(68)	a	(69)	b	(70)	d

Very Short Answer Type Questions
(अति लघु उत्तरीय प्रश्न)

Evaluate (मान ज्ञात कीजीए)

1. $\lim_{x\to 2}\left(\frac{x^5-32}{x^3-8}\right)$
2. $\lim_{x\to 2}\left(\frac{3x^2-x-10}{x^2-4}\right)$
3. $\lim_{x\to\pi}\left(\frac{\sin(\pi-x)}{\pi(\pi-x)}\right)$
4. $\lim_{x\to 0}\left(\frac{\cos 2X-1}{\cos X-1}\right)$
5. $\lim_{x\to 0}\left(\frac{(1+x)^n-1}{x}\right)$
6. $\lim_{x\to 0}\left(\frac{\sqrt{1+3x}-\sqrt{1-3x}}{x}\right)$
7. $\lim_{x\to 0}\left(\frac{\cos x}{\pi-x}\right)$
8. $\lim_{x\to 0}\left(\frac{e^{3x}-1}{2x}\right)$
9. $\lim_{x\to 0}\left(\frac{x^2-2x+1}{x+1}\right)$
10. $\lim_{x\to\pi}\left(x-\frac{22}{7}\right)$
11. $\lim_{x\to 3}\left(\frac{x^2+9}{x+3}\right)$

12. $\lim_{x\to 0}\left(\frac{\sin 7x}{8x}\right)$

13. $\lim_{x\to 0}\left(\frac{\sin 8x}{\tan 2x}\right)$

14. $\lim_{x\to 0}\left(\frac{x+1}{x-1}\right)$

15. $\lim_{x\to 4}\left(\frac{4x+3}{x-2}\right)$

16. $\lim_{x\to 1}\left(\frac{x^{10}-1}{x-1}\right)$

17. $\lim_{x\to 0}\left(\frac{x^5+2x-8}{x-8}\right)$

18. $\lim_{x\to 0}\left(\frac{ax+b}{cx+1}\right)$

19. $\lim_{x\to 0}\left(\frac{2x+1}{5}\right)$

20. $\lim_{x\to 0}\left(\frac{5^x-1}{x}\right)$

Very Short Answer Type Questions
(अति लघु उत्तरीय प्रश्न)

1. Given, $\lim_{x\to 2}\left(\frac{x^5-32}{x^3-8}\right)$, form $= \frac{0}{0}$

$= \lim_{x\to 2}\left(\frac{x^5-2^5}{x^3-2^3}\right)$

$= \lim_{x\to 2}\left(\frac{\frac{x^5-2^5}{x-2}}{\frac{x^3-2^3}{x-2}}\right)$

$= \frac{5\times 2^4}{3\times 2^2}$

$= \frac{5\times 2^2}{3} = \frac{20}{3}$ Ans

2. Given,

$\lim_{x\to 2}\frac{3x^2-x-10}{x^2-8}$

$= \frac{3(2)^2-(2)-10}{(2)^2-8}$

$= \frac{12-2-10}{4-8}$

$= \frac{12-12}{-4}$

$= \frac{0}{-4}$

$= 0$ Ans

3. Given,

$\lim_{x\to \pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}$

Let $\pi - x = h$ $\therefore h \to 0$

$= \lim_{h\to 0}\frac{\sin h}{\pi h} = \frac{1}{\pi}\lim_{h\to 0}\frac{\sin h}{h}$

$= \frac{1}{\pi}\cdot(1)$ $= \frac{1}{\pi}$ Ans

4. Given, $\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$, form $= \frac{0}{0}$

$= \lim_{x\to 0}\frac{1-\cos 2x}{1-\cos x}$

$= \lim_{x\to 0}\frac{2\sin^2 x}{2\sin^2\frac{x}{2}}$

$= \lim_{x\to 0}\left(\frac{\sin x}{\sin\frac{x}{2}}\right)^2$

$= \lim_{x\to 0}\left(\frac{\sin x}{x}\times\frac{x/2}{\sin x/2}\times 2\right)^2$

$= (1\times 1\times 2)^2$

$= 4$ Ans

5. Given, $\lim_{x\to 0}\left(\frac{(1+x)^n-1}{x}\right)$, form $= \frac{0}{0}$

let $1+x = h$

as $x \to 0$ $\therefore h \to 1$

$= \lim_{h\to 1}\frac{h^n-1^n}{h-1} = n\cdot 1^{n-1} = n$ Ans

6. Gevin,

$\lim_{x\to 0}\frac{\sqrt{1+3x}-\sqrt{1-3x}}{x}$, form $= \frac{0}{0}$

$= \lim_{x\to 0}\frac{\sqrt{1+3x}-\sqrt{1-3x}}{x}\times\frac{\sqrt{1+3x}+\sqrt{1-3x}}{\sqrt{1+3x}+\sqrt{1-3x}}$

$= \lim_{x\to 0}\frac{1+3x-1+3x}{x\cdot\left(\sqrt{1+3x}+\sqrt{1-3x}\right)}$

$= \lim_{x\to 0}\frac{6x}{x\cdot\left(\sqrt{1+3x}+\sqrt{1-3x}\right)}$

$= \lim_{x\to 0}\frac{6}{\left(\sqrt{1+3x}+\sqrt{1-3x}\right)}$

$= \frac{6}{\sqrt{1+0}+\sqrt{1-0}}$

$= \frac{6}{1+1}$

$= \frac{6}{2} = 3$ Ans

7. Given,

$$\lim_{x\to 0}\frac{\cos x}{\pi - x}$$

$$= \frac{\cos 0}{\pi - 0} = \frac{1}{\pi} \quad \text{Ans}$$

8. Given,

$$\lim_{x\to 0}\frac{e^{3x}-1}{2x}, \qquad \text{form} = \frac{0}{0}$$

$$= \lim_{x\to 0}\frac{e^{3x}-1}{3x}\times\frac{3}{2}$$

$$= 1\times\frac{3}{2} = \frac{3}{2} \quad \text{Ans}$$

9. Given,

$$\lim_{x\to 0}\frac{x^2-2x+1}{x+1}$$

$$= \frac{0^2-2\cdot(0)+1}{0+1} = 1 \quad \text{Ans}$$

10. Given,

$$\lim_{x\to \pi}\left(x-\frac{22}{7}\right)$$

$$= \pi - \frac{22}{7} \qquad \text{Ans}$$

11. Given

$$\lim_{x\to 3}\frac{x^2+9}{x+3}$$

$$= \frac{3^2+9}{3+3}$$

$$= \frac{9+9}{9}$$

$$= \frac{18}{9} = 2 \quad \text{Ans}$$

12. Given,

$$\lim_{x\to 0}\frac{\sin 7x}{8x}, \qquad \text{form} = \frac{0}{0}$$

$$= \lim_{x\to 0}\frac{\sin 7x}{7x}\times\frac{7}{8}$$

$$= 1\times\frac{7}{8} = \frac{7}{8} \quad \text{Ans}$$

13. Given,

$$\lim_{x\to 0}\left(\frac{\sin 8x}{\tan 2x}\right), \qquad \text{form} = \frac{0}{0}$$

$$= \lim_{x\to 0}\left(\frac{\sin 8x}{8x}\times\frac{2x}{\tan 2x}\times\frac{8}{2}\right)$$

$$= \left(1\times 1\times\frac{8}{2}\right) = 4 \quad \text{Ans}$$

14. Given,

$$\lim_{x\to 0}\frac{x+1}{x-1}$$

$$= \frac{0+1}{0-1} = -1 \quad \text{Ans}$$

15. Given,

$$\lim_{x\to 4}\frac{4x+3}{x-2}$$

$$= \frac{4(4)+3}{4-2} = \frac{19}{2} \quad \text{Ans}$$

16. Given,

$$\lim_{x\to 1}\frac{x^{10}-1}{x-1}, \qquad \text{form} = \frac{0}{0}$$

$$= \lim_{x\to 1}\frac{x^{10}-1^{10}}{x-1}$$

$$= 10\times 1^9$$

$$= 10\times 1 = 10 \quad \text{Ans}$$

17. Given,

$$\lim_{x\to 0}\frac{x^5+2x-8}{x-8}$$

$$= \frac{0^5+2\cdot(0)-8}{0-8}$$

$$= \frac{-8}{-8} = 1 \quad \text{Ans}$$

18. Given,

$$\lim_{x\to 0}\frac{ax+b}{cx+1}$$

$$= \frac{a\cdot(0)+b}{c\cdot(0)+1} = b \quad \text{Ans}$$

19. Given,

$$\lim_{x\to 0}\frac{2x+1}{5}$$

$$= \frac{2\cdot(0)+1}{5} = \frac{1}{5} \quad \text{Ans}$$

20. Given,

$$\lim_{x\to 0}\frac{5^x-1}{x}, \qquad \text{form} = \frac{0}{0}$$

$$= \log 5 \quad \text{Ans}$$

Short Answer Type Questions
(लघु उत्तरीय प्रश्न)

Evaluate(मान ज्ञात करें)

1. $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$
2. $\lim_{x\to 0}\left(\frac{1-\cos 4x}{1-\cos 5x}\right)$
3. $\lim_{x\to 0}\left(\frac{ax+x\cos x}{b\sin x}\right)$
4. $\lim_{x\to 0}\left(\frac{x\cdot\tan x}{1-\cos x}\right)$

5. $\lim\limits_{x\to\frac{\pi}{2}}(\sec x-\tan x)$

6. $\lim\limits_{x\to 0}\left(\dfrac{\sin ax+bx}{ax+\sin bx}\right)$

7. Find the value of $\lim\limits_{x\to 0} f(x)$, where $f(x)=\dfrac{\sin 5x}{10x}$

$\lim\limits_{x\to 0} f(x)$ का मान ज्ञात करें,जहाँ $f(x)=\dfrac{\sin 5x}{10x}$

8. Find the value of $\lim\limits_{x\to 0} f(x)$, where $f(x)=\begin{Bmatrix}2x+3, & x\le 0\\ 3(x+1), & x>0\end{Bmatrix}$

$\lim\limits_{x\to 0} f(x)$ का मान ज्ञात करें,जहाँ $f(x)=\begin{Bmatrix}2x+3, & x\le 0\\ 3(x+1), & x>0\end{Bmatrix}$

9. $\lim\limits_{x\to\frac{\pi}{2}}\left(\dfrac{\cos x}{\frac{\pi}{2}-x}\right)$

10. $\lim\limits_{x\to\pi}\left(\dfrac{1+\cos x}{\tan^2 x}\right)$

Short Answer Type Questions
(लघु उत्तरीय प्रश्न)

1. Given,

$$\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right),\qquad \text{form}=\frac{0}{0}$$
$$=\lim_{x\to 0}\frac{2\sin^2 x/2}{x^2}$$
$$=\lim_{x\to 0}2\left(\frac{\sin x/2}{x/2}\times\frac{1}{2}\right)^2$$
$$=2\left(1\times\frac{1}{2}\right)^2=2\frac{1}{4}=\frac{1}{2}\quad \text{Ans}$$

2. Given,

$$\lim_{x\to 0}\frac{1-\cos 4x}{1-\cos 5x},\qquad form=\frac{0}{0}$$
$$=\lim_{x\to 0}\frac{2\sin^2 2x}{2\sin^2 5x/2}$$
$$=\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\times\frac{5x/2}{\sin 5x/2}\times 2\times\frac{2}{5}\right)^2$$
$$=\left(1\times 1\times 2\times\frac{2}{5}\right)^2=\frac{16}{25}\quad \text{Ans}$$

3. Given,

$$\lim_{x\to 0}\left(\frac{ax+x\cdot\cos x}{b\sin x}\right),\qquad \text{form}=\frac{0}{0}$$
$$=\lim_{x\to 0}\left(\frac{a+\cos x}{b}\right)\times\frac{x}{\sin x}$$
$$=\left(\frac{a+\cos 0}{b}\right)\times(1)$$
$$=\frac{a+1}{b}\quad \text{Ans}$$

4. Given,

$$\lim_{x\to 0}\left(\frac{x\cdot\tan x}{1-\cos x}\right),\qquad \text{form}=\frac{0}{0}$$
$$=\lim_{x\to 0}\frac{x\tan x}{2\sin^2 x/2}$$
$$=\frac{1}{2}\lim_{x\to 0}x\times\frac{\tan x}{x}\times x\times\left(\frac{x/2}{\sin x/2}\times\frac{2}{x}\right)^2$$
$$=\frac{1}{2}\lim_{x\to 0}x^2\left(\frac{\tan x}{x}\right)\cdot\left(\frac{x/2}{\sin x/2}\right)^2\cdot\frac{4}{x^2}$$
$$=\frac{1}{2}(1)\cdot(1)^2\cdot 4=2\quad \text{Ans}$$

5. Given,

$$\lim_{x\to\frac{\pi}{2}}(\sec x-\tan x)$$
$$=\lim_{x\to\frac{\pi}{2}}\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)$$
$$=\lim_{x\to\frac{\pi}{2}}\left(\frac{1-\sin x}{\cos x}\right)$$
$$=\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\cos x}\times\frac{1+\sin x}{1+\sin x}$$
$$=\lim_{x\to\frac{\pi}{2}}\frac{1-\sin^2 x}{\cos x(1+\sin x)}$$
$$=\lim_{x\to\frac{\pi}{2}}\frac{\cos^2 x}{\cos x(1+\sin x)}$$
$$=\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{1+\sin x}$$
$$=\frac{\cos\frac{\pi}{2}}{1+\sin\frac{\pi}{2}}=\frac{0}{1+1}=0\quad \text{Ans}$$

6. Given,

$$\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}, \quad \text{form}=\frac{0}{0}$$

$$=\lim_{x\to 0}\left(\frac{\frac{\sin ax}{ax}\times ax+bx}{ax+\frac{\sin bx}{bx}\times bx}\right)$$

$$=\lim_{x\to 0}\left(\frac{\frac{\sin ax}{ax}\times a+b}{a+\frac{\sin bx}{bx}\times b}\right)$$

$$=\frac{1\times a+b}{a+1\times b}=\frac{a+b}{a+b}=1 \quad \text{Ans}$$

7. Given,

$$f(x)=\frac{\sin 5x}{10x}$$

$$\text{Now } \lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{\sin 5x}{10x}, \quad \text{form}=\frac{0}{0}$$

$$=\lim_{x\to 0}\frac{\sin 5x}{5x}\times\frac{1}{2}$$

$$=1\times\frac{1}{2}=\frac{1}{2} \quad \text{Ans}$$

8. Given,

$$f(x)=\begin{cases}2x+3, & x\le 0\\ 3(x+1), & x>0\end{cases}$$

$$\text{Now, } \lim_{x\to 0^+} f(x)=\lim_{h\to 0} f(0+h)=\lim_{h\to 0} 3\cdot(h+1)=3$$

$$\text{and, } \lim_{x\to 0^-} f(x)=\lim_{h\to 0} f(0-h)=\lim_{h\to 0}[2(-h)+3]=3$$

$$\text{clearly, } \lim_{x\to 0^+} f(x)=\lim_{x\to 0^-} f(x)=3$$

$$\text{Hence, } \lim_{x\to 0} f(x)=3 \quad \text{Ans}$$

9. Given,

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{\frac{\pi}{2}-x}\right), \quad \text{form}=\frac{0}{0}$$

let $\frac{\pi}{2}-x=h$

as $x\to\frac{\pi}{2} \quad \therefore h\to 0$

$$=\lim_{h\to 0}\frac{\cos\left(\frac{\pi}{2}-h\right)}{h}$$

$$=\lim_{h\to 0}\frac{\sin h}{h}=1 \quad \text{Ans}$$

10. Given,

$$\lim_{x\to\pi}\left(\frac{1+\cos x}{\tan^2 x}\right), \quad \text{form}=\frac{0}{0}$$

$$=\lim_{x\to\pi}\frac{\cos^2 x(1+\cos x)}{\sin^2 x}$$

$$=\lim_{x\to\pi}\frac{\cos^2 x\cdot(1+\cos x)}{1-\cos^2 x}$$

$$=\lim_{x\to\pi}\frac{\cos^2 x\cdot(1+\cos x)}{(1-\cos x)(1+\cos x)}$$

$$=\lim_{x\to\pi}\frac{\cos^2 x}{1-\cos x}$$

$$=\frac{\cos^2\pi}{1-\cos\pi}=\frac{(-1)^2}{1-(-1)}=\frac{1}{1+1}=\frac{1}{2} \quad \text{Ans}$$

Long Answer Type Questions
(दीर्घ उत्तरीय प्रश्न)

Evaluate (मान ज्ञात करें)

1. $\lim_{x\to\frac{\pi}{2}}\left(\frac{\tan 2x}{x-\frac{\pi}{2}}\right)$

2. $\lim_{x\to 0}\left(\frac{\sin 2x+\sin 6x}{\sin 5x-\sin 3x}\right)$

3. Let $f(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$

Find $\lim_{x\to 0} f(x)$

माना $f(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$

$\lim_{x\to 0} f(x)$ का मान ज्ञात करे।

4. Let $f(x)=\begin{cases}a+bx, & x<1\\ 4, & x=1\\ b-ax, & x>1\end{cases}$

If $\lim_{x\to 1} f(x)=f(1)$, find the value of a and b.

माना $f(x)=\begin{cases}a+bx, & x<1\\ 4, & x=1\\ b-ax, & x>1\end{cases}$

यदि $\lim_{x\to 1} f(x)=f(1)$ है तो a तथा b ज्ञात करे।

5. Show that $\lim_{x\to 0}\frac{x}{|x|}$ does not exist.

सिध्द करें कि $\lim_{x\to 0}\frac{x}{|x|}$ अस्तित्व नहीं हैं।

Long Answer Type Questions
(दीर्घ उत्तरीय प्रश्न)

1. Given,

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\tan 2x}{x-\frac{\pi}{2}}\right), \quad \text{form} = \frac{0}{0}$$

Let $x-\frac{\pi}{2}=h$

as $x\to\frac{\pi}{2}$

$\therefore h\to 0$

$$\text{Now, } \lim_{h\to 0}\frac{\tan 2\left(\frac{\pi}{2}+h\right)}{h}$$
$$=\lim_{h\to 0}\frac{\tan(\pi+2h)}{h}$$
$$=\lim_{h\to 0}\frac{\tan 2h}{h}$$
$$=\lim_{h\to 0}\left(\frac{\tan 2h}{2h}\right)\times 2$$
$$=(1)\times 2$$
$$=2 \quad \text{Ans}$$

2.Given,

$$\lim_{x\to 0}\left(\frac{\sin 2x+\sin 6x}{\sin 5x-\sin 3x}\right), \quad \text{form} = \frac{0}{0}$$
$$=\lim_{x\to 0}\left(\frac{2\sin 4x\cdot\cos 2x}{2\cos 4x\cdot\sin x}\right)$$
$$=\lim_{x\to 0}\left(\frac{\sin 4x\cdot\cos 2x}{\cos 4x\cdot\sin x}\right)$$
$$=\lim_{x\to 0}\left(\frac{\sin 4x}{\sin x}\times\frac{\cos 2x}{\cos 4x}\right)$$
$$=\lim_{x\to 0}\left(\frac{\sin 4x}{4x}\times 4x\times\frac{x}{\sin x}\times\frac{1}{x}\times\frac{\cos 2x}{\cos 4x}\right)$$
$$=\lim_{x\to 0}\left(\frac{\sin 4x}{4x}\times\frac{x}{\sin x}\times 4\times\frac{\cos 2x}{\cos 4x}\right)$$
$$=1\times 1\times 4\times\frac{1}{1}$$
$$=4 \quad \text{Ans}$$

3. Given,

$$f(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$$

We have,

$$\lim_{x\to 0^+}f(x)=\lim_{h\to 0}f(0+h)$$
$$=\lim_{h\to 0}\frac{|h|}{h}$$
$$=\lim_{h\to 0}\frac{h}{h}$$
$$=1$$
$$\text{and } \lim_{x\to 0^-}f(x)=\lim_{h\to 0}f(0-h)$$
$$=\lim_{h\to 0}\frac{|-h|}{-h}$$
$$=\lim_{h\to 0}\frac{h}{-h}$$
$$=-1$$

clearly,(स्पष्टतः)

$$\lim_{x\to 0^+}f(x)\neq\lim_{x\to 0^-}f(x)$$

so, $\lim_{x\to 0}f(x)$ does not exist.

अतः $\lim_{x\to 0}f(x)$ अस्तित्व नही है।

4. Given,

$$f(x)=\begin{cases}a+bx, & x<1\\ 4, & x=1\\ b-ax, & x>1\end{cases}$$
$$f(1)=4$$
$$\lim_{x\to 1^+}f(x)=\lim_{h\to 0}f(1+h)$$
$$=\lim_{h\to 0}[b-a(1+h)]$$
$$=b-a$$
$$\text{and } \lim_{x\to 1^-}f(x)=\lim_{h\to 0}f(1-h)$$
$$=\lim_{h\to 0}[a+b(1-h)]$$
$$=a+b$$
$$\text{Now, } \lim_{x\to 1}f(x)=f(1)=4$$
$$\lim_{x\to 1^+}f(x)=\lim_{x\to 1^-}f(x)=f(1)$$
$$\Rightarrow b-a=a+b=4$$
$$\Rightarrow b-a=4 \quad \text{and} \quad a+b=4$$
$$\Rightarrow a=0, b=4$$
$$\text{so, } a=0 \text{ and } b=4 \quad \text{Ans}$$

5. Given,

Let $f(x)=\frac{x}{|x|}$. Then,

$$\lim_{x\to 0^+}f(x)=\lim_{h\to 0}f(0+h)$$
$$=\lim_{h\to 0}\frac{h}{|h|}$$
$$=\lim_{h\to 0}\frac{h}{h}$$
$$=1$$

and $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h)$

$$= \lim_{h \to 0} \frac{-h}{|-h|}$$

$$= \lim_{h \to 0} \frac{-h}{h}$$

$$= -1$$

clearly,(स्पष्टतः)

$$\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$$

Hence, $\lim_{x \to 0} f(x)$ does not exist.

अतः $\lim_{x \to 0} f(x)$ अस्तित्व नही है।

DIFFERENTIATION
अवकलन

Derivatives (अवकलज)

1. Differentiate with respect to x (d.w.r. to x)$=\frac{d}{dx}$

   x के सापेक्ष अवकलज$=\frac{d}{dx}$

2. Differentiation from the first principle

   प्रथम सिध्दांत से अवकलन

$$y = f(x)$$

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

3. Algebra of derivative of funtions

   फलनों के अवकज का बीजगणित

   If u and v are funtions of x then

   यदी u तथा v x के फलन हैं तो

i. $\frac{d}{dx}(u+v) = \frac{du}{dx}+\frac{dv}{dx}$

ii. $\frac{d}{dx}(u-v) = \frac{du}{dx}-\frac{dv}{dx}$

iii. $\frac{d}{dx}(u\cdot v) = u\frac{dv}{dx}+v\frac{du}{dx}$

iv. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

v. $\frac{d}{dx}cu = c\cdot\frac{du}{dx}$, $c =$ constant. (नियत)

4. Some important Derivatives Furmula

   अवकलज के कुछ महत्वपूर्ण सूत्र

i. $\frac{d}{dx}c = 0$, $c =$ constant.

ii. $\frac{d}{dx}(x^n) = nx^{n-1}$

iii. $\frac{d}{dx}(x) = 1$

iv. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

v. $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

vi. $\frac{d}{dx}(a^x) = a^x\cdot\log a$

vii. $\frac{d}{dx}(e^x) = e^x$

viii. $\frac{d}{dx}(\log x) = \frac{1}{x}$

ix. $\frac{d}{dx}(\sin x) = \cos x$

x. $\frac{d}{dx}(\cos x) = -\sin x$

xi. $\frac{d}{dx}(\tan x) = \sec^2 x$

xii $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

xiii. $\frac{d}{dx}(\sec x) = \sec x\cdot\tan x$

xiv. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x\cdot\cot x$

Multiple Choice Questions
(बहु विकल्पीय प्रश्न)

1. $\frac{d}{dx}(x^{10}) =$

(a) $10x^9$ (b) x^9

(C) $10x^8$ (d) x^{10}

2. $\frac{d}{dx}(2\sqrt{x}) =$

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{1}{\sqrt{x}}$

(c) $\frac{2}{\sqrt{x}}$ (d) $\sqrt{x}$

3. $\frac{d}{dx}(5e^x) =$

(a) e^x (b) $\frac{5}{e^x}$

(c) $5e^x$ (d) None of these

4. $\frac{d}{dx}(5x) =$

(a) 5 (b) $5x$

(c) x (d) 0

5. $\frac{d}{dx}(8) =$

(a) 8 (b) 1

(c) -8 (d) 0

6. $\frac{d}{dx}(2^x) =$

(a) 2^x (b) $2^x\cdot\log 2$

(c) $\log 2$ (d) 0

7. $\frac{d}{dx}\left(\frac{1}{x}\right) =$

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$

(c) $\frac{-1}{x^2}$ (d) x^2

8. $\frac{d}{dx}(x^{-2}) =$

(a) x^{-3} (b) $-2x^{-3}$

(c) $-2x^{-1}$ (d) $2x$

9. $\frac{d}{dx}(2\log x) =$

(a) $\frac{2}{x}$ (b) $\frac{1}{x}$

(c) $2x$ (d) x

10. $\frac{d}{dx}(x^{3/2}) =$

(a) $\frac{3}{2}x^{1/2}$ (b) $x^{1/2}$

(c) $\frac{3}{2}x^{5/2}$ (d) $\frac{3}{2}x$

11. $\frac{d}{dx}(2x+8)=$

(a) 0 (b) 1

(c) 2 (d) 8

12. $\frac{d}{dx}(x^2+4x+1)=$

(a) $2x+1$ (b) $2x+4$

(c) $2x$ (d) $4x+2$

13. $\frac{d}{dx}(3x^2-8)=$

(a) $6x^2$ (b) 6

(c) $6x$ (d) $3x$

14. $\frac{d}{dx}(e^x+1)=$

(a) e^x (b) e^{-x}

(c) $x \cdot e^x$ (d) $x \cdot e^{x-1}$

15. If $y=5x^2-2x+11$ then $\frac{dy}{dx}=$

यदि $y=5x^2-2x+11$ तो $\frac{dy}{dx}=$

(a) $5x-2$ (b) $10x-2$

(c) $10x$ (d) $10x+2$

16. If $y=\sin x+\cos x$ then $\frac{dy}{dx}=$

यदि $y=\sin x+\cos x$ तो $\frac{dy}{dx}=$

(a) $\cos x-\sin x$ (b) $\sin x+\cos x$

(c) $2\sin x$ (d) $2\cos x$

17. If $y=e^x-x$ then $\frac{dy}{dx}=$

(a) e^x (b) e^x+1

(c) e^x-1 (d) -1

18. If $y=x^2-2$ then $\frac{dy}{dx}=$

(a) $2x$ (b) $2x-1$

(c) 2 (d) 0

19. If $y=99x$ then $\frac{dy}{dx}=$

(a) 100 (b) 99

(c) $99x$ (d) 0

20. If $y=x^{2023}$ then $\frac{dy}{dx}=$

(a) $2023x^{2023}$ (b) $2023x^{2022}$

(c) $2023x^{2021}$ (d) $2023x^{2024}$

21. If $y=x^{-7}+8$ then $\frac{dy}{dx}=$

(a) x^{-7} (b) $-7x^{-6}$

(c) $-7x^{-8}$ (d) x^{-8}

22. If $y=x^{\frac{5}{2}}+x^{\frac{3}{2}}+x$ then $\frac{dy}{dx}=$

(a) $\frac{5}{2}x^{3/2}+\frac{3}{2}\sqrt{x}+1$

(b) $x^{3/2}+\sqrt{x}+1$

(c) $\frac{5}{2}x^{3/2}+\sqrt{x}+1$

(d) None of these.

23. $\frac{d}{dx}e^{4x}=$

(a) e^{4x}_{4x} (b) $4 \cdot e^{4x}$

(c) $\frac{e}{4}$ (d) e^{-4x}

24. $\frac{d}{dx}\sin 5x=$

(a) $\cos 5x$ (b) $-\cos 5x$

(c) $5\cos 5x$ (d) $5\sin 5x$

25. $\frac{d}{dx}\sqrt{5x}=$

(a) $\frac{1}{2\sqrt{5x}}$ (b) $2\sqrt{5x}$

(c) $\frac{2}{5\sqrt{x}}$ (d) $\frac{5}{2\sqrt{5x}}$

26. $\frac{d}{dx}\tan 2x=$

(a) $2\cot 2x$ (b) $2\sec^2 2x$

(c) $2\tan 2x$ (d) $\sec^2 2x$

27. $\frac{d}{dx}e^{-x}=$

(a) $-e^{-x}$ (b) e^{-x}

(c) e^x (d) $-e^x$

28. $\frac{d}{dx}\log 5x=$

(a) $\frac{1}{5x}$ (b) $\frac{1}{x}$

(c) $\frac{5}{x}$ (d) None of these.

29. $\frac{d}{dx}\log(\sin x)=$

(a) $\tan x$ (b) $-\tan x$

(c) $\cot x$ (d) $-\cot x$

30. $\frac{d}{dx}\log(\cos x)=$

(a) $\tan x$ (b) $-\tan x$

(c) $\cot x$ (d) $-\cot x$

31. $\frac{d}{dx}\sqrt{\sin x}=$

(a) $\frac{1}{2\sqrt{\sin x}}$ (b) $2\sqrt{\sin x}$

(c) $\frac{\sin x}{2\sqrt{\cos x}}$ (d) $\frac{\cos x}{2\sqrt{\sin x}}$

32. $\frac{d}{dx}\sqrt{x+2}=$

(a) $2\sqrt{x+2}$ (b) $\frac{1}{2\sqrt{x+2}}$

(c) $\frac{1}{\sqrt{x+2}}$ (d) None of these.

33. $\frac{d}{dx}\log(2x+1)=$

(a) $\frac{2}{2x+1}$ (b) $\frac{1}{2x+1}$

(c) $2(2x+1)$ (d) $\frac{-1}{2x+1}$

34. $\frac{d}{dx}x\cdot e^x=$

(a) $(x+1)$ (b) $e^x\cdot(x+1)$

(c) e^x+1 (d) e^x

35. $\frac{d}{dx}x\cdot\sin x=$

(a) $x\sin x+\cos x$ (b) $x\cos x-\sin x$

(c) $x\cos x+\sin x$ (d) $x\sin x-\cos x$

36. $\frac{d}{dx}x\cdot\log x$

(a) $1+\log x$ (b) $\log x$

(c) $1-\log x$ (d) $\log x-1$

37. **The derivative of the function $\sin(x^2+1)$ w.r. to x is**
फलन $\sin(x^2+1)$ का अवकलन गुणांक के सापेक्ष होगा

(a) $\cos(x^2+1)$ (b) $2x\cos(x^2+1)$

(c) $\cos(2x^3+2x)$ (d) $\sin(x^2+1)$

38. **The derivative of the function $\cos x^3$ w.r. to x is**
फलन $\cos x^3$ का अवकलन गुणांक x के सापेक्ष होगा

(a) $3x^2\sin x^3$ (b) $-3x^2\sin x^3$

(c) $x^2\sin x^3$ (d) None of these.

39. **The value of $\frac{d}{dx}e^{x^4}$ will be**

$\frac{d}{dx}e^{x^4}$ का मान होगा

(a) $4x^3\cdot e^{x^4}$ (b) $x^3\cdot e^{x^4}$

(c) $4x^3\cdot e^{x^3}$ (d) e^{x^4}

40. **The value of $\frac{d}{dx}(3x^2-6x)$ will be**

$\frac{d}{dx}(3x^2-6x)$ का मान होगा

(a) $3x-6$ (b) $6x-6$

(c) $6x$ (d) $x-1$

41. $\frac{d}{dx}(x^2\cdot\sin x)=$

(a) $2x\sin x$ (b) $2x\sin x+x^2\cos x$

(c) $2x\cos x+x^2\sin x$ (d) $2x\cos x$

42. $\frac{d}{dx}\sin^2 x=$

(a) $\sin 2x$ (b) $\sin x$

(c) $2\sin x$ (d) $\cos 2x$

43. $\frac{d}{dx}\sin x\cdot\cos x=$

(a) $\cos x$ (b) $\sin x$

(c) $\cos 2x$ (d) $\sin 2x$

44. $\frac{d}{dx}\left(\frac{x}{x+1}\right)=$

(a) $\frac{1}{(x+1)^2}$ (b) $\frac{x}{(x+1)^2}$

(c) $\frac{-1}{(x+1)^2}$ (d) $\frac{-x}{(x+1)^2}$

45. **If $f(x)=2x^2+3x-5$ then $f'(0)+3\cdot f'(-1)=$**
यदि $f(x)=2x^2+3x-5$ तो $f'(0)+3\cdot f'(-1)=$

(a) 2 (b) 0

(c) 1 (d) -1

46. **If $f(x)=\sin x$ then $f'\left(\frac{\pi}{2}\right)=$**

यदि $f(x)=\sin x$ तो $f'\left(\frac{\pi}{2}\right)=$

(a) 2 (b) 0

(c) 1 (d) -1

47. **If $f(x)=2x^2-2x+1$ then $f'(0)+f'(2)=$**

$f(x)=2x^2-2x+1$ तो $f'(0)+f'(2)=$

(a) 2 (b) 4

(c) -2 (d) 0

48. $\frac{d}{dx}\left(\frac{\sin x}{x}\right)=$

(a) $\frac{\sin x-\cos x}{x^2}$ (b) $\frac{\sin x-x\cos x}{x^2}$

(c) $\frac{x\sin x-\cos x}{x^2}$ (d) None of these.

49. $\frac{d}{dx}\sin\sqrt{x}=$

(a) $\frac{\sin\sqrt{x}}{2\sqrt{x}}$ (b) $\frac{\cos\sqrt{x}}{\sqrt{x}}$

(c) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$ (d) $\cos\sqrt{x}$

50. $\frac{d}{dx}\tan x^2=$

(a) $\sec x^2$ (b) $2x\tan x^2$

(c) $2x\sec x$ (d) $2x\sec^2 x^2$

51. If $y=\cos\sqrt{x}$ then value of $\frac{dy}{dx}$ will be

यदि $y=\cos\sqrt{x}$ तो $\frac{dy}{dx}$ का मान होगा

(a) $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$ (b) $\frac{\sin\sqrt{x}}{2\sqrt{x}}$

(c) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$ (d) None of these.

52. If $f(x)=4x^3-2x+8$ then $f'(0)=$
यदि $f(x)=4x^3-2x+8$ तो $f'(0)=$

(a) 10 (b) 12
(c) -2 (d) 8

53. If $f(x)=e^x\sin x$ then $f'(x)$ will be
यदि $f(x)=e^x\sin x$ तो $f'(x)$ का मान होगा

(a) $e^x(\sin x+\cos x)$ (b) $e^x(\sin x-\cos x)$
(c) $e^x(\cos x-\sin x)$ (d) None of these.

54. $\frac{d}{dx}x^2\cdot e^x=$

(a) $e^x(x+2)$ (b) $e^x(x^2+2x)$
(c) $e^x(x^2+1)$ (d) $e^x(x^2-2x)$

55. $\frac{d}{dx}e^{\cos x}=$

(a) $-e^{\cos x}$ (b) $-\sin x\cdot e^{\cos x}$
(c) $\cos x\cdot e^{\cos x}$ (d) None of these.

56. $\frac{d}{dx}(5+7x)^6=$

(a) $6(5+7x)^5$ (b) $7(5+7x)^6$
(c) $42(5+7x)^5$ (d) $42(5+7x)^7$

57. $\frac{d}{dx}\sec x=$

(a) $\operatorname{cosec}x$ (b) $\sec x\cdot\cot x$
(c) $\sec x\cdot\tan x$ (d) None of these.

58. If $f(x)=\tan x$ then $f'(0)=$
यदि $f(x)=\tan x$ तो $f'(0)=$

(a) 0 (b) -1
(c) 2 (d) 1

59. $\frac{d}{dx}x^{-5/2}=$

(a) $\frac{5}{2}x^{-7/2}$ (b) $\frac{-5}{2}x^{-7/2}$

(c) $\frac{-5}{2}x^{-3/2}$ (d) None of these.

60. $\frac{d}{dx}\cos^2 x=$

(a) $-\sin 2x$ (b) $\sin 2x$
(c) $2\sin 2x$ (d) $\cos 2x$

Answer key उत्तरमाला

Multiple Choice Questions
(बहु विकल्पीय प्रश्न)

(1)	a	(2)	b	(3)	c	(4)	a	(5)	d
(6)	b	(7)	c	(8)	b	(9)	a	(10)	a
(11)	c	(12)	b	(13)	c	(14)	a	(15)	b
(16)	a	(17)	c	(18)	a	(19)	b	(20)	b
(21)	c	(22)	a	(23)	b	(24)	c	(25)	d
(26)	b	(27)	a	(28)	b	(29)	c	(30)	b
(31)	d	(32)	b	(33)	a	(34)	b	(35)	c
(36)	a	(37)	b	(38)	b	(39)	a	(40)	b
(41)	b	(42)	a	(43)	c	(44)	a	(45)	b
(46)	b	(47)	b	(48)	d	(49)	c	(50)	d
(51)	a	(52)	c	(53)	a	(54)	b	(55)	b
(56)	c	(57)	c	(58)	d	(59)	b	(60)	a

Very Short Answer Type Questions
(अति लघु उत्तरीय प्रश्न)

Find $\frac{dy}{dx}$ ($\frac{dy}{dx}$ ज्ञात करें)

1. $y=x^{-3}$
2. $y=x^{1/3}$
3. $y=2x^2+4x+1$
4. $y=10x$
5. $y=x^3-27$
6. $y=(x-1)(x-2)$
7. $y=\tan x+\cot x$
8. $y=\sin x+e^x$
9. $y=(5x+1)^2$
10. $y=\sin 3x$
11. $y=\sqrt{3x}$
12. $y=\sqrt{\tan x}$
13. $y=\cos 9x$
14. $y=x^3-4x+8$
15. $y=\frac{1}{x}$
16. $y=\frac{1}{x^2}$
17. $y=\sec x-\tan x$
18. $y=5\sin x+6\tan x$
19. $y=6x^2-12x-2023$
20. $y=5x-9x^2$
21. $y=x+x^2+x^3$
22. $y=e^{3x}$
23. $y=2x$
24. $y=e^{-x}$
25. $y=x\cdot\tan x$
26. $y=x^{-3}+2x+1$
27. $y=\operatorname{cosec}x-\sqrt{x}$
28. $y=\log x-2e^x$
29. $y=x^{3/2}-2\sqrt{x}+7$
30. $y=\log 4x$

Very Short Answer Type Questions
(अति लघु उत्तरीय प्रश्न)

1. Given, $y=x^{-3}$

d.w.r. to x

$$\frac{dy}{dx} = -3x^{-4} \quad \text{Ans}$$

2. Given, $y = x^{1/3}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{1}{3}x^{-2/3} \quad \text{Ans}$$

3. Given, $y=2x^2+4x+1$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(2x^2 + 4x + 1)$$
$$= 2\frac{dx^2}{dx} + 4 \cdot \frac{dx}{dx} + \frac{d(1)}{dx}$$
$$= 2 \cdot (2x) + 4 \cdot (1) + (0)$$
$$= 4x + 4 \quad \text{Ans}$$

4. Given, $y=10x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}10x$$
$$= 10 \cdot \frac{dx}{dx}$$
$$= 10 \cdot (1)$$
$$= 10 \quad \text{Ans}$$

5. Given, $y=x^3-27$

d.w.r. to x

$$\frac{dy}{dx} = 3x^2 - 0 = 3x^2 \quad \text{Ans}$$

6. Given, $y=(x-1)\cdot(x-2)$

$y=x^2-2x-x+2$

$y=x^2-3x+2$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 - 3x + 2)$$
$$= 2x - 3 + 0$$
$$= 2x - 3 \quad \text{Ans}$$

7. Given, $y=\tan x+\cot x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\tan x + \frac{d}{dx}\cot x$$
$$= \sec^2 x - \operatorname{cosec}^2 x \quad \text{Ans}$$

8. Given, $y=\sin x+e^x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x + e^x)$$
$$= \frac{d}{dx}\sin x + \frac{d}{dx}e^x$$
$$= \cos x + e^x \quad \text{Ans}$$

9. Given, $y=(5x+1)^2$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(5x + 1)^2$$
$$= \frac{d}{d(5x+1)}(5x + 1)^2 \times \frac{d}{dx}(5x + 1)$$
$$= 2 \cdot (5x + 1) \times (5)$$
$$= 10(5x + 1) \quad \text{Ans}$$

10. Given, $y = \sin 3x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\sin 3x$$
$$= \frac{d}{d(3x)}(\sin 3x) \times \frac{d}{dx}(3x)$$
$$= \cos 3x \times 3 = 3\cos 3x \quad \text{Ans}$$

11. Given, $y=\sqrt{3x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{3x}$$
$$= \frac{d}{d(3x)}(\sqrt{3x}) \times \frac{d}{dx}3x$$
$$= \frac{1}{2\sqrt{3x}} \times 3$$
$$= \frac{3}{2\sqrt{3x}} \quad \text{Ans}$$

12. Given, $y=\sqrt{\tan x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{\tan x}$$
$$= \frac{d}{d(\tan x)}(\sqrt{\tan x}) \times \frac{d}{dx}\tan x$$
$$= \frac{1}{2\sqrt{\tan x}} \times \sec^2 x$$
$$= \frac{\sec^2 x}{2\sqrt{\tan x}} \quad \text{Ans}$$

13. Given, $y=\cos 9x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\cos 9x$$
$$= \frac{d}{d(9x)}\cos 9x \times \frac{d}{dx}9x$$
$$= (-\sin 9x) \times (9)$$
$$= -9\sin 9x \quad \text{Ans}$$

14. Given, $y=x^3-4x+8$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}(x^3-4x+8)$$
$$=3x^2-4 \quad \text{Ans}$$

15. Given, $y=\frac{1}{x}$

d.w.r. to x

$$\frac{dy}{dx}=\frac{-1}{x^2} \quad \text{Ans}$$

16. Given, $y=\frac{1}{x^2}$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}x^{-2}$$
$$=-2x^{-3} \quad \text{Ans}$$

17. Given, $y=\sec x-\tan x$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}\sec x-\frac{d}{dx}\tan x$$
$$=\sec x\cdot\tan x-\sec^2 x$$
$$\sec x\cdot(\tan x-\sec x) \quad \text{Ans}$$

18. Given, $y=5\sin x+6\tan x$

d.w.r. to x

$$\frac{dy}{dx}=5\cdot\frac{d\sin x}{dx}+6\cdot\frac{d\tan x}{dx}$$
$$=5\cos x+6\sec^2 x \quad \text{Ans}$$

19. Given, $y=6x^2-12x-2023$

d.w.r. to x

$$\frac{dy}{dx}=6\cdot\frac{dx^2}{dx}-12\cdot\frac{dx}{dx}-\frac{d}{dx}(2023)$$
$$=12x-12-0$$
$$=12(x-1) \quad \text{Ans}$$

20. Given, $y=5x-9x^2$

d.w.r. to x

$$\frac{dy}{dx}=5\frac{dx}{dx}-9\frac{dx^2}{dx}$$
$$=5\cdot(1)-9(2x)$$
$$=5-18x \quad \text{Ans}$$

21. Given, $y=x+x^2+x^3$

d.w.r. to x

$$\frac{dy}{dx}=1+2x+3x^2 \quad \text{Ans}$$

22. Given, $y=e^{3x}$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}e^{3x}$$
$$=\frac{d}{d(3x)}e^{3x}\times\frac{d}{dx}3x$$
$$=e^{3x}\cdot 3=3e^{3x} \quad \text{Ans}$$

23. Given , $y=2x$

d.w.r. to x

$$\frac{dy}{dx}=2\cdot\frac{dx}{dx}=2(1)=2 \quad \text{Ans}$$

24. Given, $y=e^{-x}$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}e^{-x}$$
$$=\frac{d}{d(-x)}(e^{-x})\times\frac{d}{dx}(-x)$$
$$=e^{-x}\times(-1)=-e^{-x} \quad \text{Ans}$$

25. Given, $y=x\cdot\tan x$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}(x\cdot\tan x)$$
$$=x\cdot\frac{d}{dx}\tan x+\tan x\frac{dx}{dx}$$
$$=x\sec^2 x+\tan x(1)$$
$$=x\sec^2 x+\tan x \quad \text{Ans}$$

26. Given, $y=x^{-3}+2x+1$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}x^{-3}+2\cdot\frac{dx}{dx}+\frac{d}{dx}(1)$$
$$=-3x^{-4}+2 \quad \text{Ans}$$

27. Given, $y=\operatorname{cosec}x-\sqrt{x}$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}\operatorname{cosec}x-\frac{d}{dx}\sqrt{x}$$
$$=-\operatorname{cosec}x\cdot\cot x-\frac{1}{2\sqrt{x}} \quad \text{Ans}$$

28. Given, $y=\log x-2e^x$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}\log x-2\cdot\frac{d}{dx}e^x$$
$$=\frac{1}{x}-2e^x \quad \text{Ans}$$

29. Given, $y=x^{3/2}-2\sqrt{x}+7$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}x^{3/2} - 2\frac{d}{dx}\sqrt{x} + \frac{d}{dx}(7)$$
$$= \frac{3}{2}x^{1/2} - 2\cdot\frac{1}{2\sqrt{x}} + 0$$
$$= \frac{3\sqrt{x}}{2} - \frac{1}{\sqrt{x}} \quad \text{Ans}$$

30. Given, $y=\log 4x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\log 4x$$
$$= \frac{d}{d(4x)}\log 4x \times \frac{d}{dx}(4x)$$
$$= \left(\frac{1}{4x}\right)\times 4\cdot(1)$$
$$= \frac{1}{x} \quad \text{Ans}$$

Short Answer Type Questions
(लघु उत्तरीय प्रश्न)

Find $\frac{dy}{dx}$ ($\frac{dy}{dx}$ ज्ञात करें)

1. $y=x\cdot\sec x$
2. $y=x^2\cdot\cos x$
3. $y=x^3\cdot e^x$
4. $y=x\cdot\log x$
5. $y=e^x\cdot\sin x$
6. $y=e^x\cdot\tan x$
7. $y=\frac{\sin x}{x}$
8. $y=\frac{x+1}{x}$
9. $y=\frac{x+1}{x-1}$
10. $y=\sin^3 x$
11. $y=\sin x^2$
12. $y=\sqrt{\sec x}$
13. $y=\sqrt{\cot x}$
14. $y=\sin(2x-5)$
15. $y=\log(7x+1)$
16. $y=e^{4x-8}$
17. $y=e^{8x}+8$
18. $y=\log(x^2+1)$
19. $y=\cos(5x^2+1)$
20. $y=3^x-e^{2x}$
21. $y=5\sec x+4\cos x$
22. $y=x^{-4}\cdot(3-4x^{-5})$
23. $y=x^{-3}\cdot(5+3x)$
24. $y=x^5\cdot(3-6x^{-9})$
25. $y=2x-\frac{3}{4}$
26. $y=5\sin x-6\cos x+7$
27. $y=2\tan x-7\sec x$
28. $y=x+\frac{1}{x}$
29. $y=\frac{x^2+2x+1}{x}$
30. $y=4\sqrt{x}-2$

Short Answer Type Questions
(लघु उत्तरीय प्रश्न)

1. Given, $y=x\cdot\sec x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(x\cdot\sec x)$$
$$= x\cdot\frac{d}{dx}\sec x + \sec x\cdot\frac{d}{dx}x$$
$$= x\cdot(\sec x\cdot\tan x) + \sec x\cdot(1)$$
$$= \sec x\cdot(x\cdot\tan x + 1) \quad \text{Ans}$$

2. Given, $y=x^2\cdot\cos x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(x^2\cdot\cos x)$$
$$= x^2\cdot\frac{d}{dx}\cos x + \cos x\frac{d}{dx}x^2$$
$$= x^2\cdot(-\sin x) + \cos x\cdot(2x)$$
$$= x\cdot(2\cos x - x\sin x) \quad \text{Ans}$$

3. Given, $y=x^3\cdot e^x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(x^3\cdot e^x)$$
$$= x^3\cdot\frac{d}{dx}e^x + e^x\cdot\frac{d}{dx}x^3$$
$$= x^3\cdot(e^x) + e^x\cdot(3x^2)$$
$$= x^2\cdot e^x(x+3) \quad \text{Ans}$$

4. Given, $y=x\cdot\log x$

d.w.r. to x

$$\frac{dy}{dx} = x\cdot\frac{d}{dx}\log x + \log x\cdot\frac{dx}{dx}$$
$$= x\cdot\left(\frac{1}{x}\right) + \log x\cdot(1)$$
$$= 1 + \log x \quad \text{Ans}$$

5. Given, $y=e^x\cdot\sin x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(e^x\cdot\sin x)$$
$$= e^x\cdot\frac{d}{dx}\sin x + \sin x\frac{d}{dx}e^x$$
$$= e^x\cdot(\cos x) + \sin x\cdot(e^x)$$
$$= e^x\cdot(\cos x + \sin x) \quad \text{Ans}$$

6. Given, $y=e^x \cdot \tan x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(e^x \cdot \tan x)$$

$$= e^x \cdot \frac{d}{dx}\tan x + \tan x \cdot \frac{d}{dx}e^x$$

$$= e^x(\sec^2 x) + \tan x \cdot (e^x)$$

$$= e^x(\sec^2 x + \tan x) \quad \text{Ans}$$

7. Given, $y=\frac{\sin x}{x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x}{x}\right)$$

$$= \frac{x\frac{d}{dx}\sin x - \sin x \cdot \frac{d}{dx}x}{x^2}$$

$$= \frac{x \cdot (\cos x) - \sin x \cdot (1)}{x^2}$$

$$= \frac{x \cdot \cos x - \sin x}{x^2} \quad \text{Ans}$$

8. Given, $y = \frac{x+1}{x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x+1}{x}\right)$$

$$= \frac{x \cdot \frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}x}{x^2}$$

$$= \frac{x \cdot (1+0) - (x+1) \cdot (1)}{x^2}$$

$$= \frac{x - x - 1}{x^2}$$

$$= \frac{-1}{x^2} \quad \text{Ans}$$

9. Given, $y=\frac{x+1}{x-1}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x+1}{x-1}\right)$$

$$= \frac{(x-1)\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$= \frac{(x-1)(1+0) - (x+1)(1-0)}{(x-1)^2}$$

$$= \frac{x-1-x-1}{(x-1)^2}$$

$$= \frac{-2}{(x-1)^2} \quad \text{Ans}$$

10. Given, $y=\sin^3 x$

$y=(\sin x)^3$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)^3$$

$$= \frac{d}{d(\sin x)}(\sin x)^3 \times \frac{d}{dx}(\sin x)$$

$$= 3 \cdot (\sin x)^2 \times (\cos x)$$

$$= 3\sin^2 x \cdot \cos x \quad \text{Ans}$$

11. Given, $y=\sin x^2$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\sin x^2$$

$$= \frac{d\sin x^2}{d(x^2)} \times \frac{d}{dx}(x^2)$$

$$= (\cos x^2) \times (2x)$$

$$= 2x \cdot \cos x^2 \quad \text{Ans}$$

12. Given, $y=\sqrt{\sec x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{\sec x}$$

$$= \frac{d}{d(\sec x)}\sqrt{\sec x} \times \frac{d}{dx}(\sec x)$$

$$= \frac{1}{2\sqrt{\sec x}} \times \sec x \cdot \tan x$$

$$= \frac{\sqrt{\sec x} \cdot \tan x}{2} \quad \text{Ans}$$

13. Given, $y=\sqrt{\cot x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{\cot x}$$

$$= \frac{d}{d(\cot x)}\sqrt{\cot x} \times \frac{d}{dx}\cot x$$

$$= \frac{1}{2\sqrt{\cot x}} \times (-\operatorname{cosec}^2 x)$$

$$= \frac{-\operatorname{cosec}^2 x}{2\sqrt{\cot x}} \quad \text{Ans}$$

14. Given, $y=\sin(2x-5)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d}{dx}\sin(2x-5)$$

$$= \frac{d}{d(2x-5)}\sin(2x-5) \times \frac{d}{dx}(2x-5)$$

$$= \cos(2x-5) \times (2-0)$$

$$= 2\cos(2x-5) \quad \text{Ans}$$

15. Given, $y=\log(7x+1)$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}\log(7x+1)$$
$$=\frac{d}{d(7x+1)}\log(7x+1)\times\frac{d}{dx}(7x+1)$$
$$=\frac{1}{7x+1}\times(7+0)$$
$$=\frac{7}{7x+1}\quad\text{Ans}$$

16. Given, $y=e^{4x-8}$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}e^{4x-8}$$
$$=\frac{d}{d(4x-8)}e^{4x-8}\times\frac{d}{dx}(4x-8)$$
$$=e^{4x-8}\times(4-0)$$
$$=4\cdot e^{4x-8}\quad\text{Ans}$$

17. Given, $y=e^{8x}+8$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}e^{8x}+\frac{d}{dx}(8)$$
$$=\frac{d}{d8x}e^{8x}\times\frac{d}{dx}(8x)+\frac{d}{dx}(8)$$
$$=e^{8x}\times 8+0$$
$$=8e^{8x}\quad\text{Ans}$$

18. Given, $y=\log(x^2+1)$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}\log(x^2+1)$$
$$=\frac{d}{d(x^2+1)}\log(x^2+1)\times\frac{d}{dx}(x^2+1)$$
$$=\frac{1}{x^2+1}\times(2x+0)$$
$$=\frac{2x}{x^2+1}\quad\text{Ans}$$

19. Given, $y=\cos(5x^2+1)$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}\cos(5x^2+1)$$
$$=\frac{d}{d(5x^2+1)}\cos(5x^2+1)\times\frac{d}{dx}(5x^2+1)$$
$$=-\sin(5x^2+1)\times(10x+0)$$
$$=-10x\cdot\sin(5x^2+1)\quad\text{Ans}$$

20. Given, $y=3^x-e^{2x}$

d.w.r. to x

$$\frac{dy}{dx}=\frac{d}{dx}3^x-\frac{d}{dx}e^{2x}$$
$$=3^x\cdot\log 3-2e^{2x}\quad\text{Ans}$$

21. Given, $y=5\sec x+4\cos x$

d.w.r. to x

$$\frac{dy}{dx}=5\cdot\frac{d}{dx}(\sec x)+4\cdot\frac{d}{dx}\cos x$$
$$=5\sec x\cdot\tan x-4\sin x\quad\text{Ans}$$

22. Given , $y=x^{-4}(3-4x^{-5})$

$$y=3x^{-4}-4x^{-9}$$

d.w.r. to x

$$\frac{dy}{dx}=3\cdot\frac{d}{dx}x^{-4}-4\frac{d}{dx}x^{-9}$$
$$=3\cdot(-4x^{-5})-4\cdot(-9\cdot x^{-10})$$
$$=-12\cdot x^{-5}+36x^{-10}\quad\text{Ans}$$

23. Given, $y=x^{-3}\cdot(5+3x)$

$$y=5x^{-3}+3x^{-2}$$

d.w.r. to x

$$\frac{dy}{dx}=5\cdot\frac{d}{dx}x^{-3}+3\cdot\frac{d}{dx}x^{-2}$$
$$=5\cdot(-3x^{-4})+3\cdot(-2x^{-3})$$
$$=-15x^{-4}-6x^{-3}\quad\text{Ans}$$

24. Given, $y=x^5\cdot(3-6x^{-9})$

$$y=3x^5-6x^{-4}$$

d.w.r. to x

$$\frac{dy}{dx}=3\frac{d}{dx}x^5-6\frac{d}{dx}x^{-4}$$
$$=3\cdot(5x^4)-6(-4x^{-5})$$
$$=15x^4+24x^{-5}\quad\text{Ans}$$

25. Given, $y=2x-\frac{3}{4}$

d.w.r. to x

$$\frac{dy}{dx}=2\cdot\frac{dx}{dx}-\frac{d}{dx}\left(\frac{3}{4}\right)$$
$$=2\cdot(1)-0$$
$$=2\quad\text{Ans}$$

26. Given, $y=5\sin x-6\cos x+7$

d.w.r. to x

$$\frac{dy}{dx} = 5\frac{d}{dx}\sin x - 6\cdot\frac{d}{dx}\cos x + \frac{d}{dx}(7)$$
$$= 5(\cos x) - 6(-\sin x) + (0)$$
$$= 5\cos x + 6\sin x \quad \text{Ans}$$

27. Given, $y=2\tan x-7\sec x$

d.w.r. to x

$$\frac{dy}{dx} = 2\cdot\frac{d}{dx}\tan x - 7\cdot\frac{d}{dx}\sec x$$
$$= 2\cdot(\sec^2 x) - 7\cdot(\sec x\cdot\tan x)$$
$$= \sec x\cdot(2\sec x - 7\tan x) \quad \text{Ans}$$

28. Given, $y= x+\frac{1}{x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d(x)}{dx} + \frac{d}{dx}\left(1/x\right)$$
$$= (1) + \left(\frac{-1}{x^2}\right)$$
$$= 1 - \frac{1}{x^2} \quad \text{Ans}$$

29. Given, $y = \frac{x^2+2x+1}{x}$

$$y = x + 2 + \frac{1}{x}$$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d(x)}{dx} + \frac{d(2)}{dx} + \frac{d}{dx}\left(1/x\right)$$
$$= (1) + (0) + \left(\frac{-1}{x^2}\right)$$
$$= 1 - \frac{1}{x^2} \quad \text{Ans}$$

30. Given, $y= 4\sqrt{x} - 2$

d.w.r. to x

$$\frac{dy}{dx} = 4\frac{d}{dx}\sqrt{x} - \frac{d(2)}{dx}$$
$$= 4\left(\frac{1}{2\sqrt{x}}\right) - (0)$$
$$= \frac{2}{\sqrt{x}} \quad \text{Ans}$$

Long Answer Type Questions
(दीर्घ उत्तरीय प्रश्न)

Find the derivative of the following functions by First principal.

निम्नलिखित फलनो का प्रथम सिध्दांत से अवकलज ज्ञात कीजिए।

1. $f(x)=\sin x$
2. $f(x)=x$
3. $f(x)=3$
4. $f(x)=10x$
5. $f(x)=e^x$
6. $f(x)=x^2$
7. $f(x)=\frac{1}{x}$
8. $f(x)=\tan x$

Long Answer Type Questions
(दीर्घ उत्तरीय प्रश्न)

1 Given, $y=f(x)=\sin x$ ---------- (1)

By First princpal प्रथम सिध्दांत से

$$\frac{dy}{dx} = \frac{d}{dx}f(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}, \qquad \text{form} = \frac{0}{0}$$
$$= \lim_{h\to 0}\frac{2\cos\left(\frac{2x+h}{2}\right)\cdot\sin\left(h/2\right)}{h}$$
$$= \lim_{h\to 0}\cos\left(x+h/2\right)\left(\frac{\sin h/2}{h/2}\right)$$
$$= \cos\left(x+\frac{0}{2}\right)\cdot(1)$$
$$= \cos x \quad \text{Ans}$$

2. Given, $y=f(x)=x$ ----------(1)

By First principal

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{(x+h)-x}{h}$$
$$= \lim_{h\to 0}\frac{h}{h}$$
$$= 1 \quad \text{Ans}$$

3. Given, $y= f(x)=3$ ----------(1)

By First principal

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{3-3}{h}$$
$$= 0 \quad \text{Ans}$$

4. Given, $y=f(x)=10x$ ----------(1)

By First principal

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{10(x+h)-10x}{h}$$

$$=\lim_{h\to 0}\frac{10h}{h}$$

$$=10 \quad \text{Ans}$$

5. Given, $y=f(x)=e^x$ ---------- (1)

By First principal

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$$

$$=\lim_{h\to 0}\left(\frac{e^h-1}{h}\right)\cdot e^x$$

$$=1\cdot e^x$$

$$=e^x \quad \text{Ans}$$

6. Given, $y=f(x)=x^2$ ----------(1)

By First principal

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2}{h}$$

$$=\lim_{h\to 0}\frac{h(2x+h)}{h}$$

$$=\lim_{h\to 0}(2x+h)=2x \quad \text{Ans}$$

7. Given, $y=f(x)=\frac{1}{x}$ ----------(1)

By First principal

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

$$=\lim_{h\to 0}\frac{x-x-h}{h(x+h)x}$$

$$=\lim_{h\to 0}\frac{-h}{h(x+h)x}$$

$$=\lim_{h\to 0}\frac{-1}{(x+h)x}$$

$$=\frac{-1}{(x+0)x}$$

$$=\frac{-1}{x^2} \quad \text{Ans}$$

8. Given, $y=f(x)=\tan x$ ----------(1)

By First principal

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\tan(x+h)-\tan x}{h}$$

$$=\lim_{h\to 0}\frac{\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin x}{\cos x}}{h}$$

$$=\lim_{h\to 0}\frac{\sin(x+h)\cdot\cos x-\sin x\cdot(\cos(x+h)}{h\cdot\cos(x+h)\cdot\cos x}$$

$$=\lim_{h\to 0}\frac{\sin(x+h-x)}{h\cdot\cos(x+h)\cdot\cos x}$$

$$=\lim_{h\to 0}\frac{\sin h}{h\cdot\cos(x+h)\cdot\cos x}$$

$$=\lim_{h\to 0}\left(\frac{\sin h}{h}\right)\times\frac{1}{\cos(x+h)\cdot\cos x}$$

$$=(1)\times\frac{1}{\cos(x+0)\cdot\cos x}$$

$$=\frac{1}{\cos x\cdot\cos x}$$

$$=\frac{1}{\cos^2 x}$$

$$=\sec^2 x \quad \text{Ans}$$